“TIQXMMI” MILLIY TADQIQOT UNIVERSITETI HUZURIDAGI FUNDAMENTAL VA AMALIY TADQIQOTLAR INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.03/31.03.2022 T/FM.10.04 RAQAMLI ILMIY KENGASH

FUNDAMENTAL VA AMALIY TADQIQOTLAR INSTITUTI

ALLOQULOV MIRZABEK ULUG‘BEK O‘G‘LI

QORA O‘RALAR MIMIKERLARINING KUZATUV XOSSALARI VA ULARNING ENERGETIKASIGA MUQOBIL MODELLAR

01.03.01 – Astronomiya
01.04.02 – Nazariy fizika

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

Toshkent – 2024

UDK 530.12:531.51

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiya avtoreferati mundarijasi

Content of the dissertation abstract of the doctor of philosophy (PhD) on physical and mathematical sciences

Оглавление автореферата диссертации доктора философии (PhD) по физико-математическим наукам

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O‘zbekiston Respublikasi Oliy ta’lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasida B2024.1.PhD/FM1027 raqami bilan ro‘yxatga olingan.

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Ilmiy rahbarlar:	**Ahmedov Bobomurat Juraevich** Fizika-matematika fanlari doktori, akademik
	Shaymatov Sanjar Ruzimurotovich Fizika-matematika fanlari doktori
	Daniele Malafarina Fizika-matematika fanlari doktori, professor
Rasmiy opponentlar:	**Abishev Medeu Erzhanovich** Fizika-matematika fanlari doktori, akademik
	Xolikov Shukirjon Sodiqovich Fizika-matematika fanlari doktori
Yetakchi tashkilot:	**Sharof Rashidov nomidagi Samarqand davlat universiteti**

Dissertatsiya himoyasi “TIQXMMI” Milliy tadqiqot universiteti huzuridagi Fundamental vaamaliy tadqiqotlar instituti huzuridagi DSc.03/31.03.2022 T/FM.10.04 raqamli Ilmiy kengashning 202_-yil “__”__________soat __ dagi majlisida bo‘lib o‘tadi. (Manzil: 100000, Toshkent shahri, Qori Niyoziy ko‘chasi 39-uy, Fundamental va amaliy tadqiqotlar instituti, 108-katta majlislar zali; tel.: 71 237-09-61.; e-mail: info@ifar.uz

Dissertatsiya bilan “TIQXMMI” Milliy tadqiqot universiteti huzuridagi Fundamental va amaliy tadqiqotlar instituti Axborot-resurs markazida tanishish mumkin (№ raqami bilan ro‘yxatga olingan). (Manzil: 100000, Toshkent shahri, Qori Niyoziy ko‘chasi, 39-uy, Fundamental va amaliy tadqiqotlar instituti, 205- kutubxona; tel.: 71 237-09-61

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B.A. Toshmatov
Ilmiy darajalar beruvchi ilmiy kengash raisi o‘rinbosari, f.-m.f.d.

Sh.N. Mardonov
Ilmiy darajalar beruvchi ilmiy kengash ilmiy kotibi, f.-m.f.d.

B.M. Narzilloyev
Ilmiy darajalar beruvchi ilmiy kengash huzuridagi ilmiy seminar raisi, f.-m.f.d.

KIRISH (Falsafa doktori (PhD) dissertatsiya annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurligi. Hozirgi kunda, zamonaviy astronomik LIGO-VIRGO va Event Horizon teleskopi yordamida olingan kuzatuvlar yaqin qo‘shaloq sistemalardagi qora o‘ralar va qo‘shaloq neytron yulduzlarining birlashishi natijasida hosil bo‘lgan gravitatsion to‘lqinlarni bevosita aniqlash va M87 galaktika markazda joylashgan o‘ta massiv qora o ‘ra tasvirini kuzatish va tahlil qilish orqali kuchli gravitatsion maydon rejimida astrofizik hodisalarning tushuntirish imkonini bermoqda. Bu kabi zamonaviy kuzatuvlar muqobil gravitatsiya nazariyalarining yangi umumlashgan yechimlarini topish va ularning parametrlarini aniq o‘lchash hamda qorong‘u modda mohiyati bilan bog‘liq bo‘lgan muammolarni tekshirishda muhim ahamiyat kasb etishi mumkin. Lekin, yuqorida ta’kidlangan astronomik kuzatuvlarga qaramay aylanuvchi qora o‘ralarning yuqori energiyali astrofizik hodisalarini hosil bo‘lish tabiati va astrofizik qora o‘ra nomzodlarining geometriyasi bilan bog‘liq parametrlariga aniq cheklov va o‘lchovlar olish kabi savollarga oydinlik kiritish dolzarbligicha qolmoqda. Shuningdek, Eynshteynning umumiy nisbiylik nazariyasi (GR) asosiy muhim nazariya bo‘lishiga qaramay qora o‘ralarning o‘ziga xos singulyarlik va qorong‘u modda muammolarini to‘la to‘kis tushuntirish imkoniyatiga ega emas. Shu sababli, bu muammolarga nisbatan fundamental tushunishga imkon beradigan istiqbolli muqobil nazariyalarni yaratish va ularning parametrlariga aniq cheklovlar olish kabi vazifalar shu jumladan yangi modellar va muqobil umumlashgan fazo-vaqt geometriyasini yaratish va ular ustida sinovlar o‘tkazish zaruriyatiga olib keladi.

Oxirgi yillarda mamlakatimizda amaliy va fundamental tadqiqotlarga e’tibor qaratilmoqda. Xususan nazariy astrofizik tadqiqotlar shular jumlasidandir. Shubhasiz, nazariy astrofizik tadqiqotlarni rivojlantirish bugungi kunning dolzarb masalalaridan biridir. 2022-2026-yillarda O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha qabul qilingan Harakatlar strategiyasida O‘zbekistonda ilm-fanni rivojlantirish uchun fundamental tadqiqotlarning asosiy yo‘nalishlari va ularni amaliyotga joriy qilish o‘z ifodasini topgan. Astrofizik obyektlarning energetikasini o‘rganish fundamental tadqiqotlar sohasidagi muhim masalalardan biri bo‘lib qolmoqda.

Ushbu ilmiy tadqiqot ishi quyidagi me’yoriy hujjatlar bilan belgilangan vazifalarga mos keladi: O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha Harakatlar strategiyasi to‘g‘risida” gi PF-4947 sonli Farmoni, O‘zbekiston Respublikasi Prezidentining 2017-yil 18-fevraldagi “Fanlar akademiyasi faoliyatini yanada takomillashtirish, ilmiy-tadqiqot faoliyatini tashkil etish, boshqarish va moliyalashtirish chora-tadbirlari to‘g‘risida”gi PQ-2789 sonli qarori va boshqalar.

Tadqiqotning Respublika fan va texnikasini rivojlantirishning asosiy ustuvor yo‘nalishlariga muvofiqligi. Tadqiqot O‘zbekiston Respublikasi fan va texnikaning ustuvor yo‘nalishlariga muvofiq amalga oshirildi: II. “Quvvat, energiya va resurslarni tejash”.

Dissertatsiya mavzusini ushbu mavzuda dissertatsiya olib borilayotgan oliy o‘quv yurtlari va ilmiy-tadqiqot muassasalarining ilmiy ishlari bilan

bog‘lash. Taqdimnoma Innovatsion rivojlanish vazirligi tomonidan moliyalashtirilgan ilmiy loyihalar doirasida bajarilgan. F-FA-2021-510 "Modifikatsiyalangan gravitatsiya nazariyasi doirasida neytron yulduzlardagi yadro moddalarini tadqiq etish".

Tadqiqotning maqsadi Qora o‘ra mimikerlarining kuzatuv xossalarini o‘rganish va ularning energetikasiga muqobil modellarni topishdir.

Tadqiqot vazifalari:

aylanuvchi Kerr-Newman-MOG qora o‘rasining Comisso-Asenjo mexanizmi orqali magnit qayta ulanishga ta’sirini tahlil qilish

qora o‘raning aylanishi tufayli ergosfera ichida doimiy ravishda sodir bo‘ladigan magnit qayta ulanish jarayoni orqali energiya olishni o‘rganish

energiya ajratib olishning energiya samaradorligini va quvvatini plazma magnitlanishini, magnit maydon yo‘nalishi, qora o‘ra spini va MOG parametrlarining funktsiyasi sifatida barcha kerakli shartlarni qo‘yish orqali o‘rganish

magnit qayta ulanish orqali energiya olish uchun plazmaning tezlashtirilgan va sekinlashtirilgan energiyalarini o‘rganish

energiya olish uchun zarur shartni qanoatlantirish uchun faza-fazo sohani o‘rganish

magnit qayta ulanish Kerr qora o‘ra holatiga nisbatan qanchalik samarali ekanligini tushunish uchun tez aylanadigan Kerr-Newman-MOG qora o‘ralarining quvvati va energiya samaradorligini o‘rganish

magnit qayta ulanish va Blandford-Znajek mexanizmlarining quvvatini solishtirish orqali tez magnit qayta ulanishda energiya olish tezligini baholash

turli mumkin bo‘lgan holatlar uchun plazma magnitlanishiga nisbatan samaradorlik darajasini tahlil qilish

Rezzolla-Zhidenko fazosida tarqaladigan neytrino lazzatlarining tebranishlarini o‘rganish va neytrinolarni aniqlash printsipial ravishda ular harakat qilgan geometriyani cheklash uchun qanday ishlatilishi mumkinligini aniqlash

Tadqiqot obyekti astrofizik kompakt obyektlar, zarralar dinamikasi, astrofizik kompakt obyektlar energetikasiga muqobil modellar hisoblanadi.

Tadqiqot predmeti bu qora o‘ra mimikerlarining kuzatuv xossalari, qora o‘ra mimikerlarining energetikasiga muqobil nazariy modellar, skalyar maydon ta’sirida gravitatsion kompakt obyektlar yaqinidagi zarralar dinamikasini o‘rganishning nazariy modellari, differensial tenglamalarni yechishning analitik va sonli usullari hisoblanadi.

Tadqiqot metodi bu nazariy fizika va astrofizika usullari, umumiy nisbiylik nazariyasi matematik apparati, hisoblash matematikasi metodlari, matematik fizikaning zamonaviy metodlari, zarrachalar harakati uchun differentsial tenglamalarni hisoblashning analitik va sonli metodlaridan iborat.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

Energiya ajratib olishning energiya samaradorligini va quvvatni plazma magnitlanishi, magnit maydon yo‘nalishi va qora o‘ra spini va MOG parametrlari funktsiyasi sifatida barcha kerakli shartlarni qo‘yish orqali o‘rganildi.

Ilk bor aylanuvchi Kerr-Newman-MOG qora o‘rasining Comisso-Asenjo mexanizmi orqali magnit qayta ulanishga ta’siri tahlil qilindi.

Magnit qayta ulanish Kerr qora o‘ra holatiga nisbatan qanchalik samarali ekanligini tushunish uchun tez aylanadigan Kerr-Newman-MOG qora o‘ralarining quvvati va energiya samaradorligi o‘rganildi.

Ilk bor magnit qayta ulanish va Blandford-Znajek mexanizmlarining quvvatini solishtirish orqali tez magnit qayta ulanishda energiya olish tezligini baholandi.

Elektrik Penrose jarayoni orqali astrofizik kompakt obyektlardan energiya ajratib olish tahlil qilingan.

Ilk bor Rezzolla-Zhidenko fazosida tarqaladigan neytrino lazzatlarining tebranishlarini o‘rganish va neytrinolarni aniqlash printsipial ravishda ular harakat qilgan geometriyani cheklash uchun qanday ishlatilishi mumkinligini aniqlandi.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

Aylanuvchi Kerr-Newman-MOG qora o‘rasining Comisso-Asenjo mexanizmi orqali magnit qayta ulanishga ta’sirini tahlil qilingan.

Energiya ajratib olishning energiya samaradorligini va quvvatni plazma magnitlanishi, magnit maydon yo‘nalishi va qora o‘ra spini va MOG parametrlari funktsiyasi sifatida barcha kerakli shartlarni qo‘yish orqali o‘rganildi.

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Tadqiqot natijalarining ishonchliligi matematik fizika, hisoblash matematikasi va relyativistik astrofizikaning zamonaviy tasdiqlangan usullarini qo‘llash orqali ta’minlanadi. Natijalar qat’iy ravishda umumiy nisbiylik vanazariy fizikaning matematik apparati doirasida olingan. Hisoblashning zamonaviy analitik va sonli usullari ham qo‘llaniladi va natijalar mavjud kuzatuv ma’lumotlari va boshqa mualliflarning natijalari bilan taqqoslanadi. Ishning tuzilgan xulosalari kompakt obyektlar astrofizikasining asosiy qoidalariga mos keladi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqot natijalarining ilmiy ahamiyati tanlangan Rezzola-Zhidenko metrikasi sferik simmetrik statik metrikalarni hammasini o‘z ichiga olganligidan iborat.

Tadqiqot natijalarining amaliy ahamiyati esa shundan iboratki, qora o‘ra mimikerlari uchun taklif qilinayotgan model yuqori energetik jarayonlarni tushuntirishda muhim rol o‘ynashi mumkin.

Tadqiqot natijalarini amalga oshirish. Qora o‘ra mimikerlarining energetikasiga muqobil modellar bo‘yicha natijalar xorijiy tadqiqotchilarning ishlarida, nufuzli xorijiy jurnallarda qo‘llanilgan (Physical Review D, Volume 110, article id. 063003 Web-Sc, IF: 5.407 va Physical Review D, Volume 109, article id. 084066 Web-Sc, IF: 5.407).

Tadqiqot natijalarining aprobatsiyasi. Tadqiqot natijalari 2 ta xalqaro konferensiya va 1 ta mahalliy ilmiy konferensiyalarda ma'ruza qilindi va muhokama qilindi.

Tadqiqot natijalarini nashr etish. PhD tadqiqot natijalari O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasi tomonidan tavsiya etilgan ilmiy jurnallarida chop etilgan 14 ta ilmiy maqolalarda taqdim etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish, uchta bob, xulosa va foydalanilgan adabiyotlar ro'yxatidan iborat bo'lib, uning hajmi 120 betni tashkil etadi.

DISSERTATSIYANING ASOSIY MAZMUNI

Dissertatsiyaning kirish qismida mavzuning dolzarbligi va zarurligi, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo'nalishlariga muvofiqligi, muammoning o'rganilganlik darajasi, dissertatsiya bajarilgan ilmiy-tadqiqot tashkilotining ilmiy-tadqiqot rejalari bilan bog'liqligi ko'rsatilgan. Ilmiy tadqiqot ishining maqsadi, vazifalari, o'rganish ob'ekti, predmeti, usullari, ilmiy yangiligi, amaliy natijasi, ishonchliligi, natijalarning ilmiy va amaliy ahamiyati, natijalarni amaliyotga joriy etilishi, natijalarning aprobatsiyasi, natijalarning nashr qilinishi, shuningdek dissertatsiyaning tuzilishi va hajmi to'grisida qisqacha ma'lumotlar keltirilgan.

Dissertatsiyaning birinchi "**Rezzolla-Zhidenko fazo-vaqtida neytrinolarning gravitatsion linzalanishi**" deb nomlangan bobida, mavzu haqida qisqacha ma'lumot, dissertatsiyaning maqsadi, qora o'ralar va ularning mimikerlari to'g'risida ma'lumotlar berilgan. Shuningdek, ushbu fazo-vaqtda neytrinolarning tebranish ehtimolliklari tahlil qilingan.

Umumiy sferik simmetrik statik metrikaning chiziqli elementini sferik koordinatalarda $\{t, r, \theta, \phi\}$ quyidagicha yozish mumkin

$$ds^2 = -N^2(r)dt^2 + \frac{B^2(r)}{N^2(r)}dr^2 + r^2 d\Omega^2 \tag{1}$$

bu yerda $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ metrikaning odatiy sferik qismi, N va B lar esa faqat radial koordinata r ning funksiyalari. Hodisalar gorizontining radial joylashuvi $r = r_0 > 0$ sifatida belgilanadi va bu ta'rif shuni anglatadiki

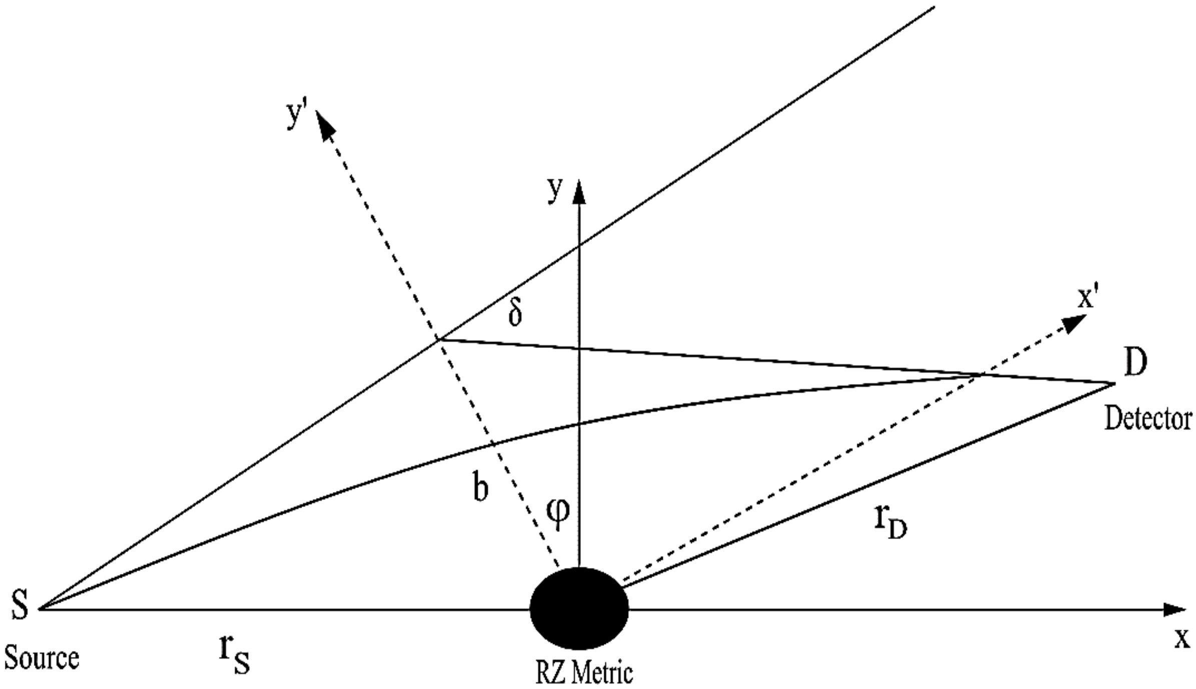
$$N^2(r_0) = 0 \tag{2}$$

Biz har qanday kosmologik effektni hisobga olmaymiz, shuning uchun chiziqli elementni (1) assimptotik tekis sifatida olish mumkin. Keyin radial koordinatani yangi o'lchamsiz kattalik x ni kiritish orqali kompakt shaklda quyidagicha yozish mumkin

$$x = 1 - \frac{r_0}{r} \quad (3)$$

Shubhasiz $x = 0$ hodisalar gorizontining joylashuviga to‘g‘ri keladi va $x = 1$ esa fazoviy cheksizlikka mos keladi. Metrik funksiya $N(r)$ ni o‘lchamsiz o‘zgaruvchi x ning funksiyasi sifatida qayta yozishimiz mumkin

$$N^2(x) = xA(x) \quad (4)$$



1-rasm. Neytrinolarning RZ fazo-vaqtida kuchsiz linzalashning sxematik diagrammasi. Neytrinolar *S* manbadan RZ fazo-vaqti bilan tavsiflangan statik va sferik massiv jismning tashqi qismidagi *D* detektorga tarqaladi.

bu yerda

$$A(x) > 0 \text{ for } 0 \le x \le 1 \quad (5)$$

Endi esa A va B funksiyalarni o‘lchamsiz kattaliklar ϵ, a_0 va b_0 larni kiritish orqali quyidagicha qayta yozishimiz mumkin

$$A(x) = 1 - \epsilon(1-x) + a_0(1-x)^2 + \tilde{A}(x)(1-x)^3, \ B(x) = 1 + b_0(1-x) + \tilde{B}(x)(1-x)^2 \quad (6)$$

bu yerda $\tilde{A}(x)$ va $\tilde{B}(x)$ funksiyalar metrikaning asimptotik xarakterini tavsiflash uchun foydalanigan.
$\tilde{A}(x)$ va $\tilde{B}(x)$ funksiyalarni cheksiz kasr ko‘rinishida yozish mumkin

$$\tilde{A}(x) = \frac{a_1}{1 + \frac{a_2 x}{1 + \frac{a_3 x}{1 + \cdots}}}, \ \tilde{B}(x) = \frac{b_1}{1 + \frac{b_2 x}{1 + \frac{b_3 x}{1 + \cdots}}} \quad (7)$$

bu yerda a_1, a_2, a_3 ... va b_1, b_2, b_3 ... lar o‘lchamsiz o‘zgarmaslardir. Keyin metrik funksiyalar istalgan tartibda qatorga yoyish mumkin. Biz uni quyidagicha bajaramiz

$$N^2(x) = 1 - (1+\epsilon)(1-x) + a_0(1-x)^2 + (a_1 - a_0 + \epsilon)(1-x)^3 - a_1(1-x)^4$$
$$B^2(x) = (1 + b_0(1-x) + b_1(1-x)^2)^2 \quad (8)$$

E'tibor berish kerakki, ϵ parametr hodisalar gorizonti radiusi r_0 ning $2M$ dan farq qilishini tavsiflaydi, chunki u hodisalar gorizontiga bog‘liq

$$\epsilon = -\left(1 - \frac{2M}{r_0}\right) \quad (9)$$

bu yerda M fazo-vaqtning ADM massasi. Gorizonti $r_0 = 2M$ bo‘lgan Schwarzschild fazovaqtini olish uchun $\epsilon = 0$ and $a_i = b_i = 0 (i = 0,1,2 \ldots)$ bo‘lishi kerak. Umumiy holda N va B funksiyalar post-Newtonian (PPN) parametrlar orqali quyidagicha yozish mumkin

$$N^2 = 1 - \frac{2M}{r} + (\beta - \gamma)\frac{2M^2}{r^2} + \mathcal{O}(1/r^3), \ \frac{B^2}{N^2} = 1 + \gamma\frac{2M}{r} + \mathcal{O}(1/r^2) \quad (10)$$

shunday qilib biz quyidagini olamiz

$$\beta - \gamma = \frac{2a_0}{(1+\epsilon)^2}, \ \gamma - 1 = \frac{2b_0}{1+\epsilon} \quad (11)$$

Parametrlash har bir perturbatsiya tartibda Schwarzschild metrikasidan chetlanishlarni tavsiflashga imkon beradi. Quyida biz sferik koordinatalarga qaytamiz va chiziqli element (1) ni quyidagicha yozamiz

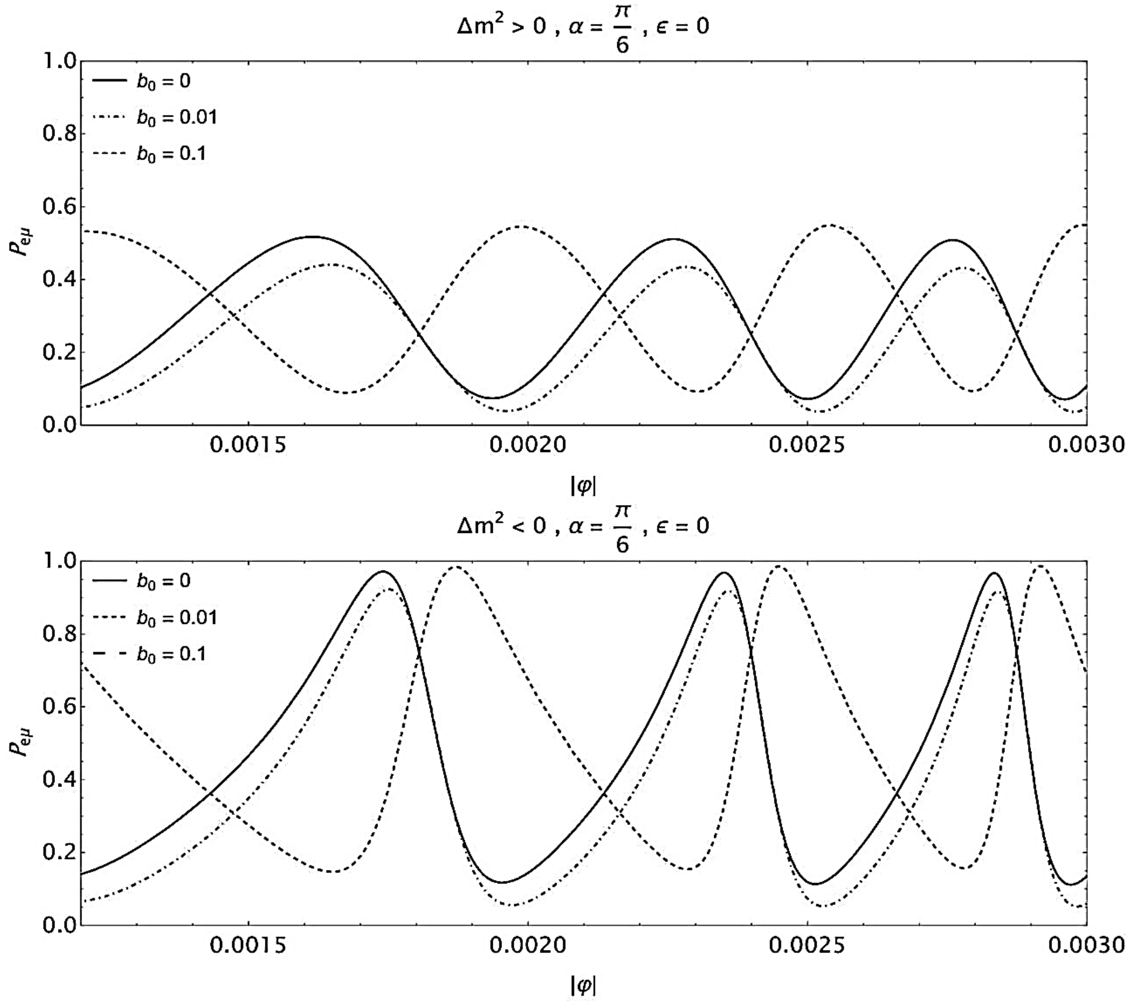
$$ds^2 = -\mathcal{A}dt^2 + \mathcal{B}dr^2 + \mathcal{C}d\theta^2 + \mathcal{D}d\phi^2 \quad (12)$$

bu yerda $\mathcal{A}$ va $\mathcal{B}$ A va B larning birinchi tartiblarini saqlab qolish orqali aniqlangan, ya’ni $\epsilon \neq 0$ and $b_0 \neq 0$ va qolgan barcha parametrlar yo‘qolib ketgan. Bu quyidagini beradi

$$\mathcal{A}(r) = N^2(r) = 1 - (1+\epsilon)\left(\frac{r_0}{r}\right), \quad \mathcal{B}(r) = \frac{B^2(r)}{N^2(r)} = \frac{\left(1 + b_0\left(\frac{r_0}{r}\right)\right)^2}{1 - (1+\epsilon)\left(\frac{r_0}{r}\right)} \quad (13)$$
$$\mathcal{C}(r) = r^2, \qquad \mathcal{D}(r) = r^2 \sin^2\theta$$

Tekis fazo-vaqtda neytrino tebranishlari

Kuchsiz ta’sirlarda neytrinolar turli xil ta’m holatlarida hosil bo‘ladi va aniqlanadi hamda quyidagicha belgilanadi $|\nu_\alpha\rangle \alpha = e, \mu, \tau$ va ta’m holatlari bu massa holatlarining superpozitsiya bo‘lib, quyidagicha belgilanadi ν_i bu yerda $i = 1,2,3$. Massa va ta’m holatlari orasidagi bog‘lanishni quyidagicha yozishimiz mumkin



2-rasm. Yuqori panel: neytrino tebranishi ehtimolligi azimutal burchak funksiyasi sifatida $b_0 = 0$ uchun (uzluksiz chiziq), $b_0 = 0.01$ (nuqtali-uzuq chiziq) va $b_0 = 0.1$ (uzuq chiziq) normal ierarxiya $\Delta m^2 > 0$ uchun. Quyi panel: neytrino tebranishi ehtimolligi $b_0 = 0$ uchun (uzluksiz chiziq), $b_0 = 0.01$ (nuqtali-uzuq chiziq) va $b_0 = 0.1$ (uzuq chiziq) teskari ierarxiya $\Delta m^2 < 0$ uchun. Bu yerda aralashtirish burchagi $\alpha = \pi/6$. Boshqa parameterlarning qiymatlari: $M = 1M_{\odot}, \Delta m^2 = 10^{-3}\mathrm{eV}^2$ va bu yerda yengil neytrinolar massasiz deb hisoblangan.

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle \tag{14}$$

bu yerda U 3×3 unitar aralashtiruv matritsasi. Uchta ta'm neytrino tebranishi uchun bu Pontecorvo-Maki-Nakagava-Sakata (PMNS) leptonik aralashtirish matritsasi sifatida tanilgan. Biz neytrino to'lqin funksiyasi tekis to'lqin deb faraz qilamiz va u fazo-vaqt hodisasi (t_S, x_S) da joylashgan S manbadan fazo-vaqt hodisasi (t_D, x_D) da joylashgan detektorga tarqaladi. Shuning uchun detektor nuqtasida to'lqin funksiyasi quyidagicha berilgan

$$|\nu_i(t_D, x_D)\rangle = \exp(-i\Phi_i)|\nu_i(t_S, x_S)\rangle \tag{15}$$

bu yerda Φ_i tebranish fazasi. Neytrinolar boshlang'ich ta'm holatida $|\nu_\alpha\rangle S$ da hosil bo'lgan va keyin D detektorga harakatlangan deb faraz qilamiz. Bu holda neytrino ta'mining ν_α dan ν_β ga o'zgarish ehtimolligi quyidagicha

$$\mathcal{P}_{\alpha\beta} = \left|\langle \nu_\beta \mid \nu_\alpha(t_D, x_D)\rangle\right|^2 = \sum_{i,j} U_{\beta i} U_{\beta j}^* U_{\alpha j} U_{\alpha i}^* \exp\left(-i(\Phi_i - \Phi_j)\right) \tag{16}$$

Agar $\Phi_i \neq \Phi_j$ bo'lsa, ta'm o'zgarishi sodir bo'lishi mumkin. Turli xil neytrino massasining holatlari ularning massasi va energiyasi/momentidagi farqlar tufayli Φ_i turli fazalarini rivojlantiradi, bu esa oxir-oqibat neytrino tebranish hodisalarini keltirib chiqaradi. Tekis fazo-vaqtda faza quyidagicha berilgan

$$\Phi_i = E_i(t_D - t_S) - \mathbf{p}_i \cdot (\mathbf{x}_D - \mathbf{x}_S) \tag{17}$$

Odatda manbada dastlab hosil qilingan ta'm holatidagi barcha massa holatlari bir xil impuls yoki energiyaga ega deb taxmin qilinadi. Ushbu farazlarning har biri $(t_D - t_S) \simeq |\mathbf{x_D} - \mathbf{x_S}|$ bilan birga relativistik neytrinolar $(E_i \gg m_i)$ uchun quyidagiga olib keladi

$$\Delta\Phi_{ij} = \Phi_i - \Phi_j \simeq \frac{\Delta m_{ij}^2}{2E_0} |x_D - x_S| \tag{18}$$

bu yerda $\Delta m_{ij}^2 = m_i^2 - m_j^2$ va E_0 esa manbada hosil bo'lgan relativistik neytrinolarning o'rtacha energiyasi. Egri fazo-vaqtda neytrino tarqalishi uchun faza Φ_i ifodasini umumiylashtirish uchun Eq.(17) kovariant shaklda yozilgan

$$\Phi_i = \int_S^D p_\mu^{(i)} dx^\mu \tag{19}$$

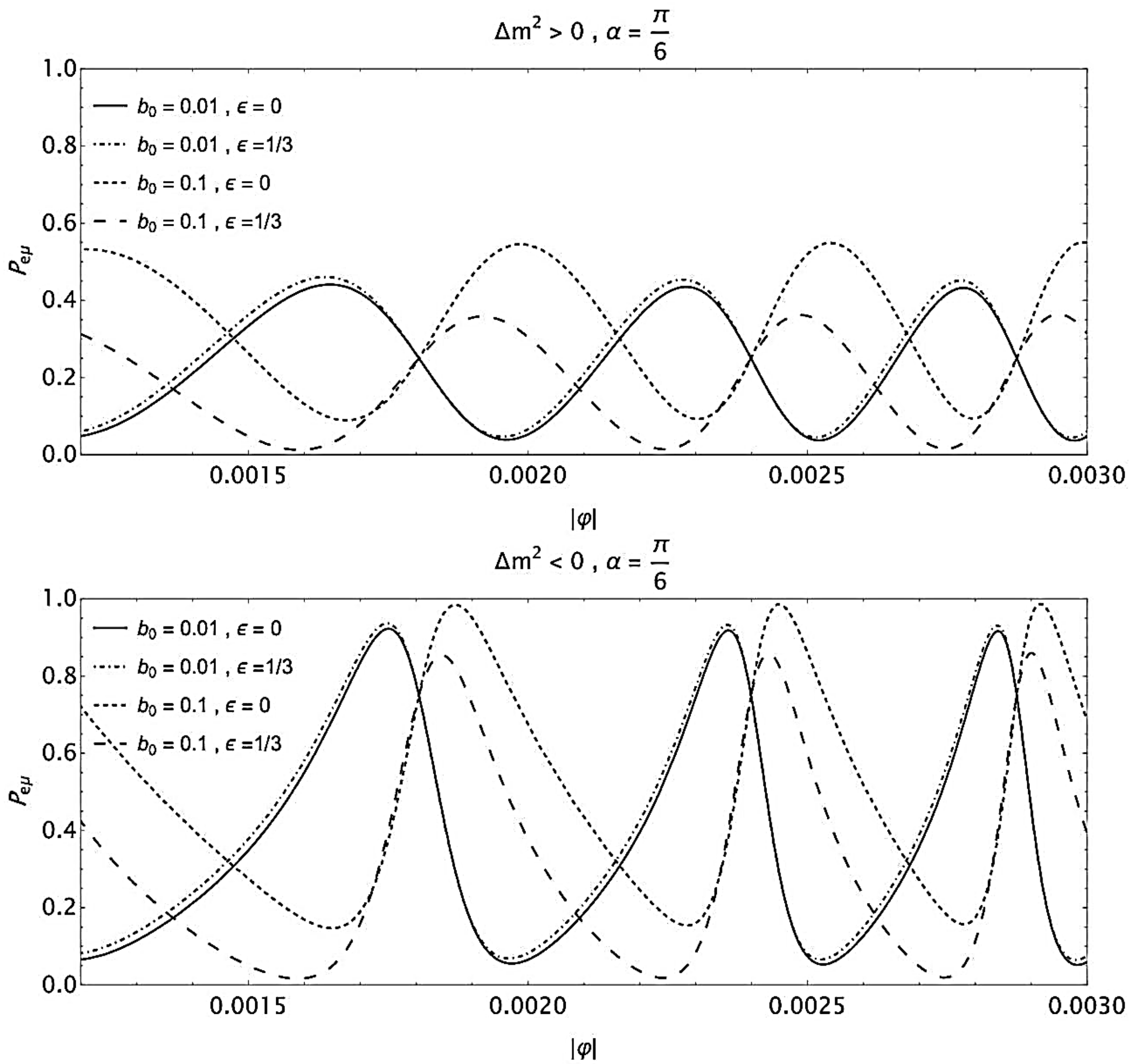
bu yerda

$$p_\mu^{(i)} = m_i g_{\mu\nu} \frac{dx^\nu}{ds} \tag{20}$$

x^ν koordinataning kanonik konjugat momenti, $g_{\mu\nu}$ va ds mos ravishda metrik tenzor va egrilangan fazo-vaqtning chiziqli elementi.

Neytrino tebranish ehtimolliklari

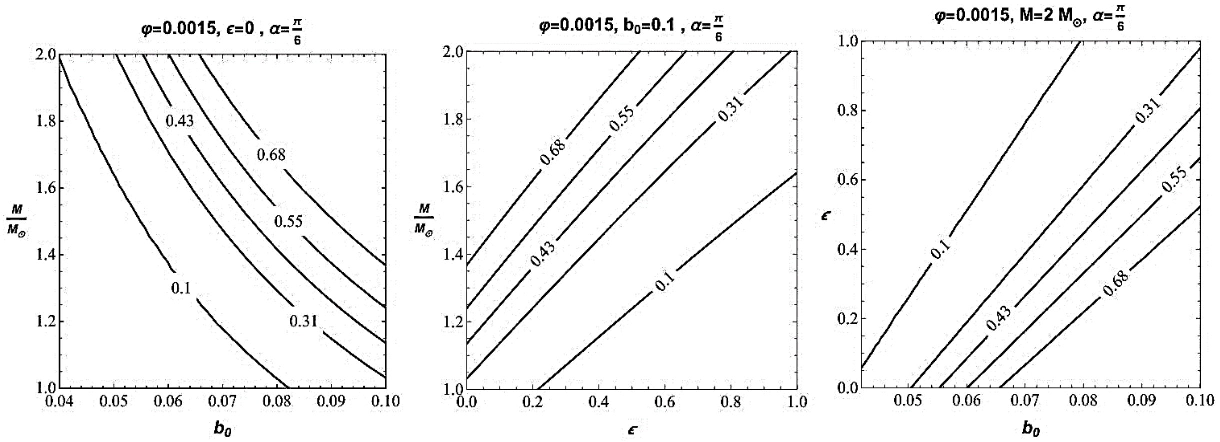
Gravitatsion linzalangan neytrino uchun tebranish fazasini olganimizdan so'ng, endi xuddi shu tebranish ehtimolligini hisoblab chiqmoqchimiz. Biz massa holati ν_i bo'lgan neytrinolarni RZ fazo-vaqtda harakat qilayotganini ko'rib chiqamiz. Asosiy g'oya shundan iboratki, S manbadan chiqarilgan neytrino ikki xil yo'l bo'ylab harakatlanishi mumkin, masalan p va q ularning tegishli masofalari har xil va D detektorida kvant interferensiyasini keltirib chiqaradi. Neytrino S manbada ta'm holatida $|\nu_\alpha, S\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$ hoil bo'ladi va quyidagiga aylanadi



3-rasm. Yuqori panel: neytrino tebranishi ehtimolligi azimutal burchak φ funksiyasi sifatida $b_0 = 0.01, \epsilon = 0$ uchun (uzluksiz chiziq), $b_0 = 0.01, \epsilon = 1/3$ (nuqtali-uzuq chiziq), $b_0 = 0.1, \epsilon = 0$ (uzuq chiziq) va $b_0 = 0.1, \epsilon = 1/3$ (katta uzuq chiziq) normal ierarxiya $\Delta m^2 > 0$ uchun. Quyi panel: neytrino tebranishi ehtimolligi azimutal burchak φ funksiyasi sifatida $b_0 = 0.01, \epsilon = 0$ uchun (uzluksiz chiziq), $b_0 = 0.01, \epsilon = 1/3$ (nuqtali-uzuq chiziq), $b_0 = 0.1, \epsilon = 0$ (uzuq chiziq) va $b_0 = 0.1, \epsilon = 1/3$ (katta uzuq chiziq) teskari ierarxiya $\Delta m^2 > 0$ uchun. Bu yerda aralashtirish burchagi $\alpha = \pi/6$. Boshqa parameterlarning qiymatlari: $M = 1M_{\odot}, \Delta m^2 = 10^{-3}\mathrm{eV}^2$.

$$|\nu_\alpha, D\rangle = N \sum_i U_{\alpha i}^* \sum_p \exp\left(-i\Phi_i^p\right)|\nu_i\rangle \tag{21}$$

bu yerda N normalizatsiya o‘zgarmasi va Φ_i^p va b_p impakt parametr bilan berilgan hamda bu p yo‘lga bog‘liq deb tushunilishi kerak. Detektorda neytrino ta’mi ν_α dan ν_β ga



4-rasm. Chap panel: Massa parametri M va RZ parametri b_0 bilan $\epsilon = 0$ bilan ma'lum $\mathcal{P}_e$ uchun ϕ o'zgarmas burchak ostida aniqlash o'rtasidagi degeneratsiya $\mathcal{P}_{e\mu}(M, b_0) = \text{const}$ dan olingan $M(b_0)$ yashirin funksiyasining $2D$ kontur grafigi bilan tasvirlangan. o'rta panel: M massa parametri va RZ parametri ϵ $\mathcal{P}_{e\mu}$ ehtimolligi uchun $b_0 = 0,1$ bilan o'zgarmas burchak ϕ ostida degeneratsiya tasvirlangan $\mathcal{P}_{e\mu}(M, \epsilon) = \text{const}$ tenglamasidan olingan $M(\epsilon)$ yashirin funksiyasining $2D$ kontur grafigi bilan tasvirlangan. o'ng panel: $\mathcal{P}_{e\mu}(\epsilon, b_0) = \text{const}$ dan olingan $\epsilon(b_0)$ yashirin funksiyasining $2D$ kontur grafigi ϵ parametri o'rtasidagi degeneratsiyani ko'rsatadi va ϕ o'zgarmas burchak ostida berilgan $\mathcal{P}_{e\mu}$ ehtimoli uchun RZ ko'rsatkichining b_0 parametri. Bu yerda M massa parametri $2M_\odot$ bo'lsin deb faraz qilinadi. Har bir egri chiziqning o'zida ko'rsatilganidek, $\mathcal{P}_{e\mu}$ o'zgarmas qiymatiga mos keladi.

o'zgarishi uchun tebranish ehtimolligi quyidagicha yozilashi mumkin

$$\mathcal{P}_{\alpha\beta} = \left|\langle \nu_\beta \mid \nu_\alpha, D\rangle\right|^2 = |N|^2 \sum_{i,j} U_{\beta i} U_{\beta j}^* U_{\alpha j} U_{\alpha i}^* \sum_{p,q} \exp\left(-i\Delta\Phi_{ij}^{pq}\right) \tag{22}$$

bu yerda

$$|N|^2 = \left(\sum_{i} |U_{\alpha i}|^2 \sum_{p,q} \exp\left(-i\Delta\Phi_{ii}^{pq}\right)\right)^{-1} \tag{23}$$

normalizatsiya o'zgarmasi va $\Delta\Phi_{ij}^{pq}$ faza o'zgarishi quyidagicha berilgan

$$\Phi_{ij}^{pq} = \Phi_i^p - \Phi_j^q = \Delta m_{ij}^2 A_{pq} + \Delta b_{pq}^2 B_{ij} \tag{24}$$

bu yerda

$$A_{pq} = \frac{r_S + r_D}{2E_0}\left[1 + \frac{2M}{r_S + r_D} + \frac{2b_0 M}{(r_S + r_D)(1+\epsilon)} \ln\frac{r_S r_D}{M^2} - \frac{\Sigma b_{pq}^2}{4 r_S r_D}\right]$$

Yuqoridagi tenglamalarda $\Delta m_{ij}^2, \Sigma m_{ij}^2, \Delta b_{pq}^2$ va Σb_{pq}^2 kattaliklar quyidagicha aniqlanadi

$$\Delta m_{ij}^2 = m_i^2 - m_j^2,\ \Sigma m_{ij}^2 = m_i^2 + m_j^2$$

RZ fazo-vaqtida neytrino linzalanishining miqdoriy qayta ishlash uchun biz ikkita neytrino ta'mining oddiy o'yinchoq modelini ko'rib chiqamiz $\nu_e \to \nu_\mu$. Ushbu o'tish uchun tebranish ehtimolligi

$$\mathcal{P}_{e\mu}^{\text{lens}} = |N|^2 \sin^2 2\alpha \left[\sin^2\left(\Delta m^2 \frac{A_{11}}{2}\right) + +\sin^2\left(\Delta m^2 \frac{A_{22}}{2}\right) - \cos(\Delta b^2 B_{12})\cos(\Delta m^2 A_{12}) + \right.$$

$$\left. + \frac{1}{2}\cos(\Delta b^2 B_{11}) + \frac{1}{2}\cos(\Delta b^2 B_{22})\right] \tag{27}$$

bo'ladi va normalizatsiya o'zgarmasi quyidagiga qisqaradi

$$|N|^2 = \frac{1}{2}\left(1 + \cos^2\alpha\cos(\Delta b^2 B_{11}) + \sin^2\alpha\cos(\Delta b^2 B_{22})\right)^{-1} \tag{28}$$

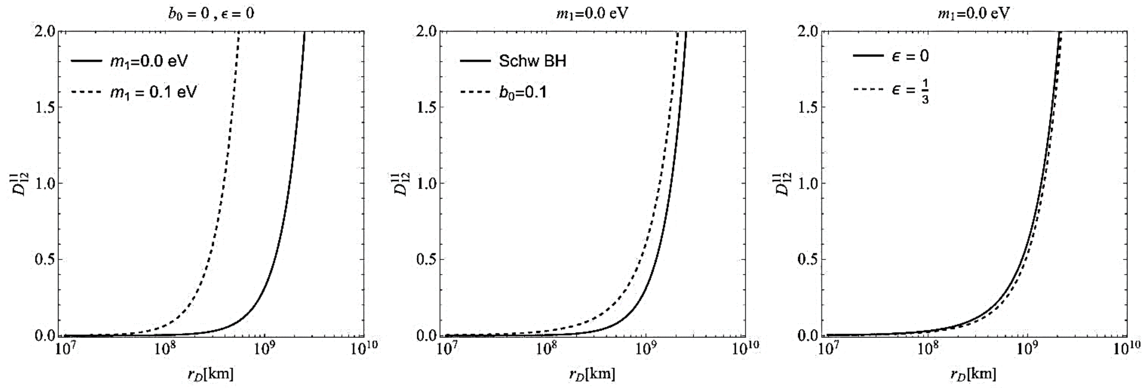
Ushbu holatda $\Delta b^2 = \Delta b_{12}^2$ va $\Delta m^2 = \Delta m_{21}^2$. E'tibor bering, Δb^2 funksiya juft bo'lgani uchun, $\mathcal{P}_{e\mu}^{\text{lens}}$ b_1 va b_2 almashganda o'zgarmaydi. Biroq, tebranish ehtimolligi neytrino massasi tartibiga sezgir bo'lib, $\Delta m^2 > 0$ va $\Delta m^2 < 0$ uchun turli natijalarga olib keladi. Bundan tashqari, tebranish ehtimolligi Schwarzschild fazo-vaqtidan va sferik simmetriyadan biroz chetga chiqishlarga sezgir ekanligini, A_{pq} ga RZ ning deformatsiya parametrlari b_0 va ϵ bog'liq hadlarni qo'shish orqali ko'rishimiz mumkin.

Ikki ta'mli o'yinchoq model uchun raqamli natijalar

Yaxshiroq tushunish uchun biz ba'zi real stsenariylarda linzalash parametrlarining o'zgarishi ehtimoliga qanday ta'sir qilishini ko'rishni istaymiz. Shuning uchun ta'sir parametrini ma'lum geometrik miqdorlarda ifodalashimiz kerak. Shu maqsadda RZ fazoda kuchsiz linzalashning sxematik diagrammasi tasvirlangan Fig 1 ga murojaat qilishimiz mumkin. Neytrinolar S nuqtasida ishlab chiqariladi va geometriyasi RZ fazo-vaqti bilan tasvirlangan massiv obyekt tomonidan linzalanadi. Oxir-oqibat neytrinolar D nuqtasida qayd qilinadi. Fig 1 dan $\{x, y\}$ Dekart tekisligida manba va detektorning massiv obyektdan masofalarini mos ravishda $r_S(x, y)$ va $r_D(x, y)$ kabi yoza olishimizni ko'ramiz. Shu bilan bir qatorda (x, y) orginal sistemani φ ga burish orqali boshqa koordinata sistemani hosil qilishimiz mumkin quyidagi kabi $x' = x\cos\varphi + y\sin\varphi$ and $y' = -x\sin\varphi + y\cos\varphi$. Aylangan ramkadagi burilish burchagi δ ni
quyidagicha deb hisoblaymiz

$$\delta \sim \frac{y_D' - b}{x_D'} = -\frac{4M}{b} = -\frac{2R_x}{b} \tag{29}$$

bu yerda $R_x = 2M = r_0(1 + \epsilon)$ va (x_D', y_D') detektorning joylashuvi. $\sin\varphi = b/r_S$ ayniyatdan foydalanib, oxirgi tenglamani quyidagicha yozish mumkin



5-rasm. $\boldsymbol{\sigma/E_{loc} = 10^{-13}}$ **uchun** $\boldsymbol{D_{12}^{11}}$ **damping faktori** $\boldsymbol{r_D}$ **ning funksiyasi sifatida. Chap panelda uzluksiz (uzuq) chiziq** $\boldsymbol{m_1 = 0\text{eV}(m_1 = 0.1\text{eV})}$ **ga mos keladi. o‘rta panelda uzluksiz va uzuq chiziqlar mos ravishda Schwarzschild va RZ chiziqli elemntlariga (** $\boldsymbol{b_0 = 0.1}$ **va** $\boldsymbol{\epsilon = 0}$ **bilan) mos keladi. o‘ng panel** $\boldsymbol{\epsilon}$ **parametrning ikkita turli qiymatlari uchun chizilgan. Bu yerda** $\boldsymbol{m_1 = 0.0\text{eV}}$ **and** $\boldsymbol{b_0 = 0.1}$ **. Qolgan parametrlar esa** $\boldsymbol{r_S = 10^5 r_D, \Delta m_{21}^2 = m_2^2 - m_1^2 = 10^{-3}\text{eV}^2}$ **va** $\boldsymbol{E_{\text{loc}} = 10\text{MeV}}$**.**

$$(2R_x x_D + b y_D)\sqrt{1 - \frac{b^2}{r_S^2}} = b^2\left(\frac{x_D}{r_S} + 1\right) - \frac{2R_x b y_D}{r_S} \qquad (30)$$

(30) tenglamaning yechimi r_S, R_x va detektorning joylashuvi (x_D, y_D) bo‘yicha impakt parametrlarini beradi. Misol tariqasida biz Quyosh-Yer tizimini geometrik miqdorlarning tipik qiymatlari bilan ko‘rib chiqamiz va Yerning joylashuvi, $r_D = 10^8$ km detektor sifatida qabul qilinganda, Quyoshning tortishish maydoni RZ metrikasi bilan ifodalanishi mumkin deb faraz qilamiz. Keyin biz manba Quyosh orqasida kattaroq masofada joylashgan deb taxmin qilamiz $r_S = 10^5 r_D$ va u odatdagi energiya $E_0 = 10\text{MeV}$ bo‘lgan relyativistik neytrinolarni chiqaradi. Endi detektorning Quyosh atrofida aylana traektoriyasini $x_D = r_D \cos\varphi$ va $y_D = r_D \sin\varphi$ deb faraz qilsak, Eq. (30) bilan berilgan kvartal ko‘phadni sonli yechishimiz va har bir φ uchun ikkita musbat haqiqiy ildiz b_1 va b_2 olishimiz mumkin. Ushbu raqamli mashqda neytrinoning tebranish ehtimoli faqat $R_x \ll b_p \ll r_D$ bo‘lgan b_p qiymati uchun hisoblanadi. Boshqacha qilib aytadigan bo‘lsak, neytrinolar RZ materiya manbasidan yetarlicha uzoqqa o‘tib, kuchsiz linzalashni hisobga olish uchun detektorning joylashuvi impakt parametridan ancha uzoqroqda olinadi. Boshqa tegishli parametrlar $M = 1M_{\odot}$ va $|\Delta m^2| = 10^{-3}\text{eV}^2$. E'tibor bering, bu raqamlar faqat tasvirlash uchun mo‘ljallangan va real stsenariyda modelning geometrik parametrlarining tegishli raqamli qiymatlari hisobga olinishi kerak.

Neytrinolarning ikki ta’mli o‘yinchoq modeli uchun tebranish ehtimoli 2 va 3 rasmlarda ko‘rsatilgan. E'tibor bering, bizning asosiy maqsadimiz tebranish ehtimolining RZ fazo-vaqti parametrlariga bog‘liqligini o‘rganishdir. Fig. 2 da neytrino tebranish ehtimolligi $\nu_e \to \nu_\mu$ Schwarzschildga mos keladidan $b_0 = 0$ (uzluksiz chiziq), $b_0 = 0.01$ (nuqtali chiziq) va $b_0 = 0.1$ (uzuq chiziq) lar uchun tasvirlangan. Rasmning yuqori panelida oddiy ierarxiya, ya’ni $\Delta m^2 > 0$, quyi panelida esa teskari ierarxiya, ya'ni $\Delta m^2 < 0$ ko'rsatilgan. Ehtimollik RZ deformatsiya parametrining b_0 qiymatiga bog'liqligini ko'rish mumkin. Bu yerda aralashtirish burchagi $\alpha = \pi/6$ ga teng. Fig. 3 da biz tebranish ehtimolligini azimutal burchak φ funksiyasi sifatida tasvirlaymiz. Yana b_0 parametri tebranish ehtimoliga qanday ta'sir qilishi aniq bo'lib, ϵ effekti faqat b_0 yetarlicha katta bo'lganda ko'rinadi.

Shunisi qiziqki, bir xil tebranish ehtimolini keltirib chiqaradigan RZ parametrlarining turli qiymatlari uchun degeneratsiya mavjud. Fig. 4 da biz $M(b_0)$ (chap panel) va $M(\epsilon)$ (o'rta panel) φ burchakning berilgan qiymatida bir xil $\mathcal{P}_{\mu\nu}$ ehtimolligini olingan funksiyalarini chizdik. Fig. 4 ning o'ng panelida ϵ va b_0 qiymatlari ko'rsatilgan, ular ma'lum bir linza massasi uchun $M = 2M_\odot$ va ma'lum burchak φ ostida bir xil ehtimollikni beradi.

Dissertatsiyaning ikkinchi "**Kerr-Newman-MOG qora o'raning magnit qayta ulanishga ta'siri**" deb nomlangan bobida, astrofizik kompakt obyektlardan energiya ajratib olishning muqobil modeli sifatida magnit qayta ulanish jarayoni haqida ma'lumotlar berilgan. Shuningdek, berilgan qora o'raning magnit qayta ulanish jaryoniga ta'siri muhokama qilingan.

Boyer-Lindquist koordinatalarida Kerr-Newman-MOG fazo-vaqt geometriyasi quyidagicha berilgan

$$ds^2 = -\frac{\Delta}{\rho^2}[dt - a\sin^2\theta d\phi]^2 + \rho^2\left[\frac{dr^2}{\Delta} + d\theta^2\right] + \frac{\sin^2\theta}{\rho^2}[(r^2 + a^2)d\phi - adt]^2 \qquad (31)$$

bu yerda

$$\rho^2 = r^2 + a^2\cos^2\theta,\ \Delta = r^2 - 2G_\mathrm{N}(1+\alpha)Mr + a^2 + Q^2 + G_\mathrm{N}^2\alpha(1+\alpha)M^2 \qquad (32)$$

MOG parametr $\alpha = (G - G_\mathrm{N})/G_\mathrm{N}$ Nyutonning gravitatsion doimiysi va qo'shimcha gravitatsion doimiy orasidagi farqni o'lchamsiz o'lchovi. Kerr-MOG qora o'rasining Arnowitt-Deser-Misner (ADM) massasi va impuls momenti quyidagicha berilgan $\mathcal{M} = (1+\alpha)M.\Delta$ funktsiyasi ADM massa bo'yicha qayta yozilishi mumkin,

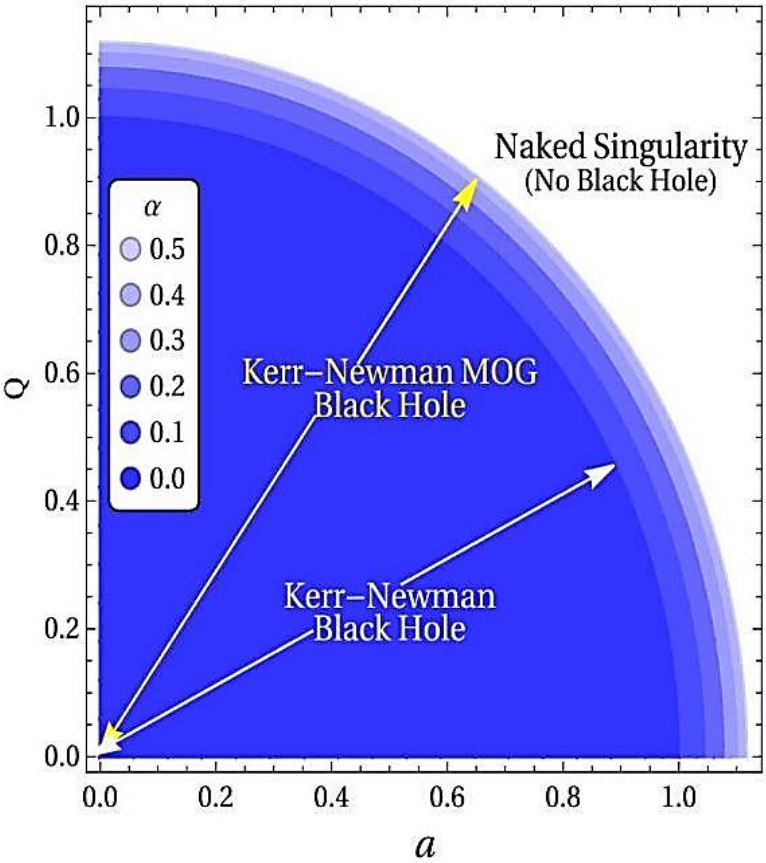
$$\Delta = r^2 - 2\mathcal{M}r + a^2 + Q^2 + \frac{\alpha}{1+\alpha}\mathcal{M}^2 \qquad (33)$$

bu yerda biz umumiylikni yo'qotmasdan $G_\mathrm{N} = 1$ o'rnatdik. Gorizontlarning fazoviy joylashuvi Δ ning ildizlari hisoblanadi

$$r_H = \mathcal{M} \pm \sqrt{\frac{\mathcal{M}^2}{1+\alpha} - a^2 - Q^2} \qquad (34)$$

E'tibor bering, agar $\mathcal{M}^2 \geq (1+\alpha)(a^2 + Q^2)$ bo'lsa, Kerr-Newman-MOG fazo-vaqti parametrlari hodisa gorizonti bilan o'ralgan qora o'rani ifodalaydi.

Bu yerda tengsizlik ekstremal qora o‘ra holatiga mos keladi, ya’ni qora o‘raning maksimal

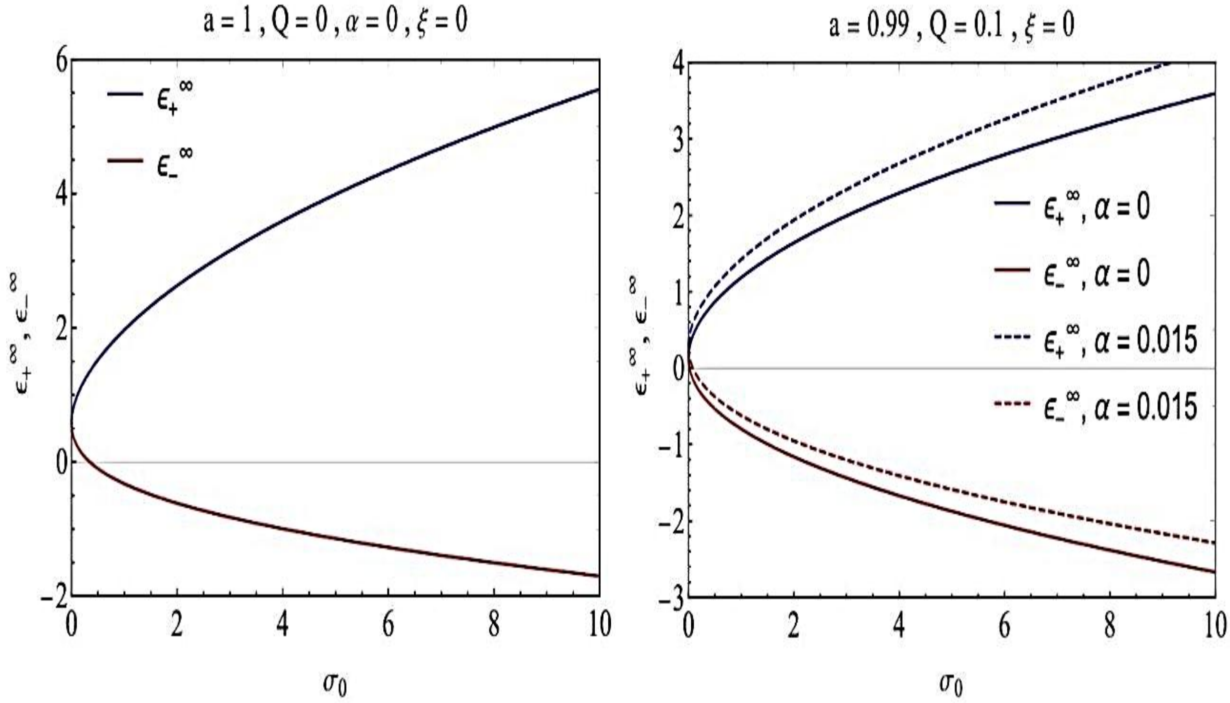


6-rasm. MOG parametr α ning 0-0.5 oraliqdagi qiymatlari uchun Kerr-Newman-MOG qora o‘rasining zaryad parametri Q va spin parametri a orasidagi parametr fazo grafigi. Bundan tashqari, M parametri birga o‘rnatilgan.

spini $a \rightarrow a_{ext}$ ga ishora qiladi. Agar bu qanoatlanmasa, unda qora o‘ra mavjud bo‘la olmaydi. Boshqa tomondan, yana bir muhim nuqta borki, Kerr-Newman-MOG qora o‘ralari MOG parametri α ta’sirida Kerr qora o‘rasidagidan kattaroq spin bilan aylanadi; Fig. 6 ga qarang.

Waldga amal qilib, Kerr-Newman-MOG qora o‘rasining atrofidagi elektromagnit maydonning vektor potentsialining umumlashtirilgan shaklini ko‘rib chiqamiz, bu quyidagi shaklda berilgan:

$$A^{\mu} = \left[\frac{Qr}{r^2 - 2f} - aB\left(1 + \frac{f_2 \sin^2\theta}{r^2}\right)\right]\xi_t^{\mu} + \left[-\frac{B}{2}\left(1 + \frac{2f_2}{r^2}\right) + \frac{Qa}{r(r^2 - 2f)}\right]\xi_{\phi}^{\mu} (35)$$



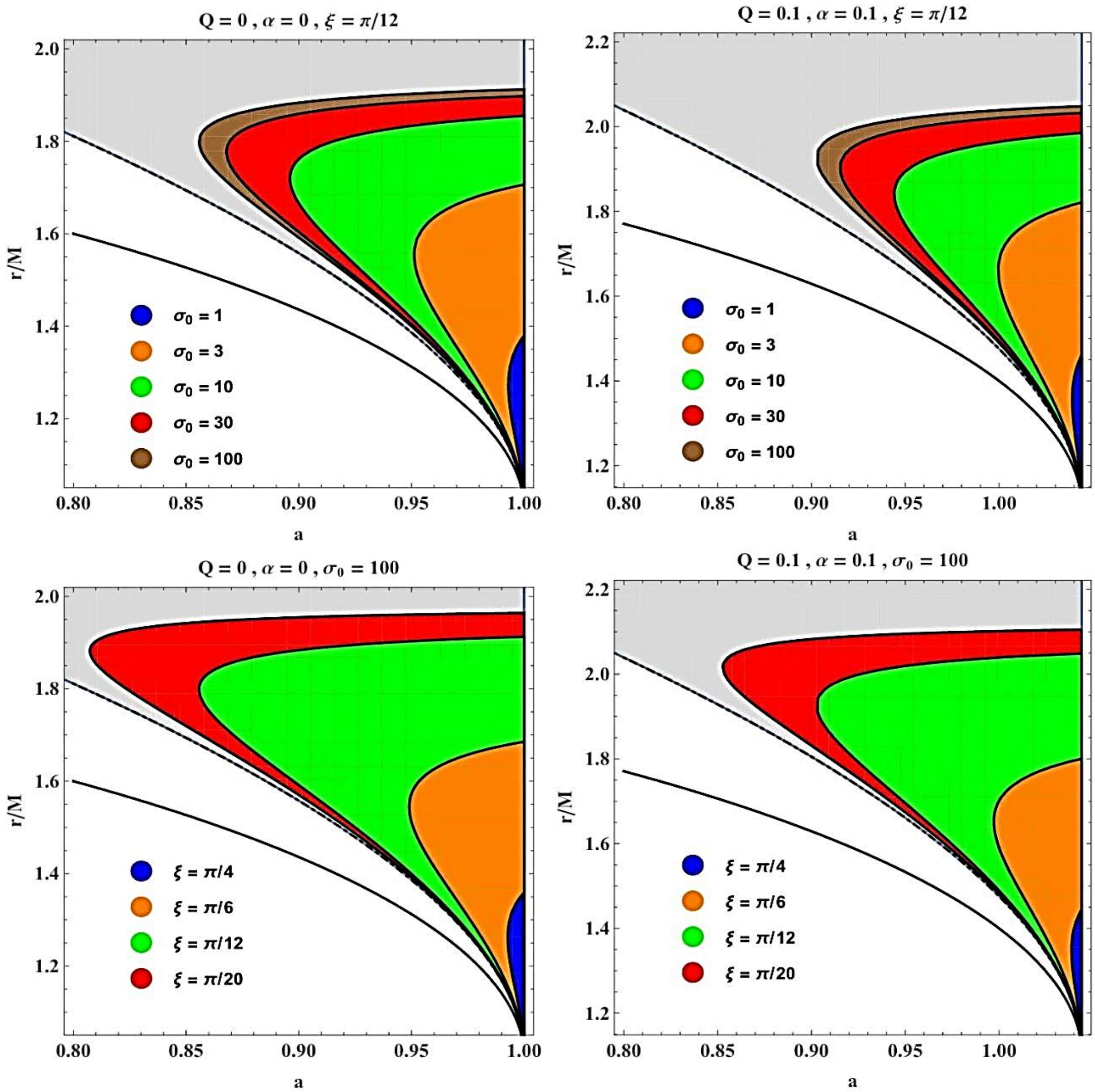
7-rasm. Har xil mumkin bo‘lgan holatlar uchun cheksizlikdagi entalpiya uchun ϵ_+va ϵ_- energiyalari chizilgan. Chapda: ϵ_+va ϵ_- $Q = 0$ va $\alpha = 0$ uchun chizilgan. o‘ngda: ϵ_+va ϵ_- α va Q o‘zgarmagan holda MOG parametrining mavjudligi natijasida chiziladi. E'tibor bering, biz maksimal energiya olish uchun ξ ni 0 ga o‘rnatdik.

bu yerda $f = f_1 r + f_2$. f_1 va f_2 quyidagicha niqlanganligini unutmang

$$f_1 = (1+\alpha)M \; f_2 = -(1+\alpha)(\alpha M^2 + Q^2)/2 \qquad (36)$$

Magnit qayta ulanish jarayoni orqali energiya olish

Qora o‘ralar eng hayratlanarli obyektlardir, chunki ularning boy astrofizik energetik hodisalari, ya’ni energiya diapazoni $E \approx 10^{42} - 10^{47}$erg/s bo‘lgan faol galaktik yadrolardan chiqadigan oqimlar rentgen nurlari, γ nurlari va juda uzoq bazaviy interferometriya kuzatuvlari yordamida kuzatilgan. Shamollar va oqimlar ko‘rinishidagi bu oqimlar qora o‘ra atrofidagi akkretsiya diskida harakatlanadigan zaryadlangan zarrachalar harakati bilan bog‘liq. Ushbu energetik hodisani o‘rganish uchun biz yaqinda taklif qilingan Komisso-Asenjo mexanizmini qo‘llash orqali magnit qayta ulash jarayonini tahlil qilamiz, bu magnit maydon chiziqlarini tez aylanadigan qora o‘ra atrofida aylantiruvchi ramkani tortish effektiga kuchli bog‘liq. Bundan tashqari, biz magnit qayta ulanish tez aylanadigan Kerr-Newman-MOG qora tuynugining ergoregionida sodir bo‘lishini hisobga olamiz, shunda uning aylanish energiyasi ushbu stsenariy bo‘yicha samarali ravishda chiqariladi. Buning uchun biz birinchi navbatda nol burchak momentum kuzatuvchisi (ZAMO) ramkasini ko‘rib chiqamiz va keyin plazma energiyasining zichligini tekshiramiz. Yuqorida qayd etilgan ZAMO ramkasi uchun chiziq elementi quyidagicha



8-rasm. Faza-fazo hududlar $\{a, r/M\}$ tezlashtirilgan plazma energiyasi $\epsilon_{+}^{\infty} > 0$ (kulrang maydon) va sekinlashtirilgan plazma energiyasi $\epsilon_{-}^{\infty} < 0$ (ko‘kdan qizilgacha bo‘lgan maydonlar) uchun chizilgan. Yuqori qator, chap/o‘ng panellar: $\{a, r/M\}$ Kerr/Kerr-Newman-MOG qora o‘ralari holida plasma magnitlanishi σ_0 ning turli kombinatsiyalari uchun chizilgan. Quyi qator, chap/o‘ng panellar: $\{a, r/M\}$ Kerr/Kerr-Newman-MOG qora o‘ralari holida orientatsiya burchagi ξ ning turli kombinatsiyalari uchun chizilgan.

yoziladi:

$$ds^2 = -d\hat{t}^2 + \sum_{i=1}^{3} \left(d\hat{x^{\iota}}\right)^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} \tag{37}$$

bu yerda biz $d\hat{t}$ va $d\hat{x^{\iota}}$ ni quyidagicha aniqlaymiz:

$$d\hat{t} = \alpha dt \text{ and } d\hat{x^{\iota}} = \sqrt{g_{ii}}dx^{i} - \alpha\beta^{\phi}dt \tag{38}$$

α va $\beta^{i} = \left(0,0,\beta^{\phi}\right)$ mos ravishda laps funksiyasi va siljish vektoriga ishora qiladi va quyidagicha yoziladi

$$\alpha = \sqrt{-g_{tt} + \frac{g_{\phi t}^2}{g_{\phi\phi}}} \text{ and } \beta^{\phi} = \frac{\sqrt{g_{\phi\phi}}\omega^{\phi}}{\alpha} \tag{39}$$

To‘g‘riroq bo‘lish uchun plazmaning bir-suyuqlik yaqinlashishi uchun energiya-momentum tensorini aniqlaymiz va u quyidagicha ifodalanadi

$$T^{\mu\nu} = pg^{\mu\nu} + wU^{\mu}U^{\nu} + F_{\delta}^{\mu}F^{\nu\delta} - \frac{1}{4}g^{\mu\nu}F^{\rho\delta}F_{\rho\delta} \tag{40}$$

E'tibor bering, yuqoridagi tenglamada p va w mos ravishda plazma bosimi va entalpiya zichligiga ishora qiladi, U^{μ} va $F^{\mu\nu}$ to‘rt tezlik va elektromagnit maydonning tenzoriga murojaat qiladi. Bu yerda biz entalpiya zichligi $w = e_{\text{int}} + p$ bilan berilganligini ta’kidlaymiz, bu yerda issiqlik energiyasining zichligi quyidagicha aniqlanadi

$$e_{int} = \frac{p}{\Gamma - 1} + \rho c^2 \tag{41}$$

Γ va ρ, mos ravishda adiabatik indeks va to‘la massa zichligiga ishora qiladi. Ushbu shartni qo‘yish orqali biz relativistik issiq plazmani holat tenglamasi bilan aniqlaymiz. Cheksizlikdagi energiya zichligi quyidagi munosabat bilan berilishi mumkin

$$e^{\infty} = -\alpha g_{\mu 0}T^{\mu 0} \tag{42}$$

Buni hisobga olib, cheksiz energiya zichligini quyidagicha yozish mumkin:

$$e^{\infty} = \alpha\hat{e} + \alpha\beta^{\phi}\hat{P}^{\phi} \tag{43}$$

Bu yerda mos ravishda $\hat{e}$ va $\hat{P}^{\phi}$ umumiy energiya zichligi va impuls zichligining azimutal komponentasini aniqlaydi hamda ular quyidagicha ifodalanadi

$$\hat{e} = w\hat{\gamma}^2 - p + \frac{\hat{B}^2 + \hat{E}^2}{2}, \ \hat{P}^{\phi} = w\hat{\gamma}^2\hat{v}^{\phi} + (\hat{B} \times \hat{E})^{\phi} \tag{44}$$

bu yerda $\hat{v}^{\phi}$ ZAMOdagi plazma tezligining azimutal komponentini ifodalaydi. Yuqoridagi tenglamada paydo bo‘ladigan Lorentz faktor $\hat{\gamma}$ va elektr $\hat{E}^{i}$ va magnit $\hat{B}^{i}$ maydon komponentlari quyidagicha aniqlanadi

$$\hat{\gamma} = \hat{U}^0 = \sqrt{1 - \sum_{i=1}^{3}(d\hat{v}^i)^2}, \ \hat{B}^i = \epsilon^{ijk}\hat{F}_{jk}/2 \text{ and } \hat{E}^i = \eta^{ij}\hat{F}_{j0} = \hat{F}_{i0} \tag{45}$$

bu yerda shuni ta’kidlash joizki, e^{∞} cheksizligidagi energiya zichligi ikki qismdan, ya’ni $e^{\infty} = e_{hyd}^{\infty} + e_{em}^{\infty}$ bo‘lgan gidrodinamik va elektromagnit qismlardan iborat, uni quyidagicha alohida yozish mumkin:

$$e_{hyd}^{\infty} = \alpha\hat{e}_{hyd} + \alpha\beta^{\phi}w\hat{\gamma}^2\hat{v}^{\phi},\; e_{em}^{\infty} = \alpha\hat{e}_{em} + \alpha\beta^{\phi}(\hat{B}\times\hat{E})_{\phi} \tag{46}$$

bu yerda $\hat{e}_{hyd} = w\hat{\gamma}^2 - p$ va $\hat{e}_{em} = \left(\hat{B}^2 + \hat{E}^2\right)/2$, mos ravishda ZAMO dagi gidrodinamik va elektromagnit maydon energiya zichliklariga ishora qiladi. Keyin cheksizlikdagi energiya zichligini baholashimiz kerak. Buning uchun biz elektromagnit maydonning hissasini chiqarib tashlash mumkin deb taxmin qilamiz, chunki uning ta’siri cheksizlikdagi gidrodinamik energiya zichligidan farqli o‘laroq juda kichikdir. Biroq, magnit maydon energiyasining katta qismi magnit qayta ulash jarayonida plazma kinetik energiyasiga o‘tkazilishi mumkinligini ta’kidlaymiz. Hammasini jamlagan holda, biz yaqinlashish uchun siqilmaydigan va adiabatik plazma uchun murojaat qilamiz deb taxmin qilamiz. Bunda cheksizlikdagi energiya zichligi quyidagi shakldа aniqlanadi:

$$e^{\infty} = e_{hyd}^{\infty} = \alpha w\hat{\gamma}\left(1 + \beta^{\phi}\hat{v}^{\phi}\right) - \frac{\alpha p}{\hat{\gamma}} \tag{47}$$

Keyinchalik, magnit qayta ulash jarayonini kichik miqyosda aniqlash uchun uni lokalizatsiya qilish kerak. Buning uchun, lokal tinch ramka $x'^{\mu} = (x'^0, x'^1, x'^2, x'^3)$ o‘yinga keladi, chunki bulk plazma qora o‘ra atrofida ekvatorial tekislikda quyidagicha aniqlanadigan Keplerian chastota/burchak tezlik bilan aylanadi

$$\Omega_K = \frac{d\phi}{dt} = \frac{-g_{t\phi,r} + \sqrt{g_{t\phi,r}^2 - g_{tt,r}g_{\phi\phi,r}}}{g_{\phi\phi,r}} \tag{48}$$

Kerr-Newman-MOG qora o‘ralari uchun Kepler chastotasi quyidagicha yoziladi:

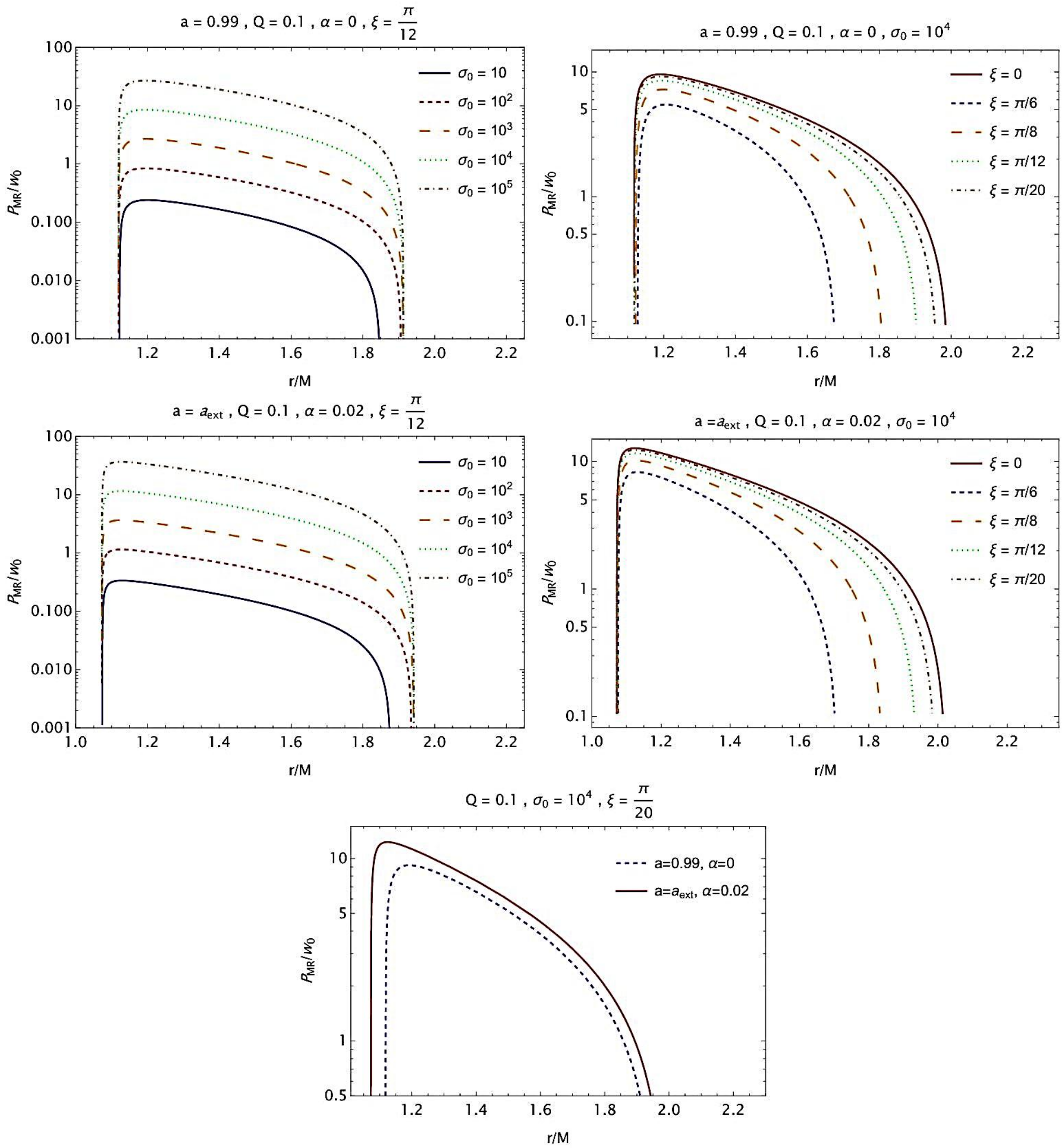
$$\Omega_K = \frac{a(Q^2 - (\alpha+1)(r-\alpha))}{r^4 + a^2(Q^2 - (\alpha+1)(r-\alpha))} + \frac{r^2\sqrt{(\alpha+1)(r-\alpha) - Q^2}}{r^4 + a^2(Q^2 - (\alpha+1)(r-\alpha))} \tag{49}$$

Shuni ta’kidlash joizki, x'^{μ} yo‘nalishi x'^1 va x'^3 mos ravishda radial $x^1 = r$ va azimutal $x^3 = \phi$ yo‘nalishlariga parallel bo‘lishi uchun tanlangan. Eq. (49) tenglamasidan keyin biz ZAMO da korotatsiya qiluvchi Kepler chastotasini ko‘rib chiqamiz va u quyidagicha berilgan

$$\hat{v}_K = \frac{d\hat{x}^{\phi}}{d\hat{x}^t} = \frac{\sqrt{g_{\phi\phi}}dx^{\phi}/d\lambda - \alpha\beta^{\phi}dx^t/d\lambda}{\alpha dx^t/d\lambda} = \frac{\sqrt{g_{\phi\phi}}}{\alpha}\Omega_K - \beta^{\phi} \tag{50}$$

Keyin Kepler chastotasini o‘rnatish orqali $\hat{v}_K$ va Lorentz faktor $\hat{\gamma}_K = 1/\sqrt{1-\hat{v}_K^2}$ shakllarini Eq. (48) bilan berilgan Kepler chastotasi Ω_K ni ta’sir qildirish orqali olish mumkin. Magnit qayta ulanish jarayoni orqali olinadigan qora o‘raning aylanish energiyasi plazma dinamikasiga va elektromagnit maydon xossalariga bog‘liq. Yuqorida aytib o‘tilganidek, biz adiabatik va siqilmaydigan plazma yaqinlashuviga bo‘ysunadigan bir suyuqlikli plazmani qabul qilamiz, shunda cheksizlikda entalpiya uchun gidrodinamik energiya quyidagicha yoziladi:

$$\epsilon_{\pm}^{\infty} = \alpha\hat{\gamma}_K \left[\left(1 + \beta^{\phi}\hat{v}_K\right)\sqrt{1+\sigma_0} \pm \cos\xi\left(\beta^{\phi} + \hat{v}_K\right)\sqrt{\sigma_0} - \frac{\sqrt{1+\sigma_0} \mp \cos\xi\,\hat{v}_K\sqrt{\sigma_0}}{4\hat{\gamma}^2\left(1+\sigma_0-\cos^2\xi\,\hat{v}_K^2\sigma_0\right)} \right] \quad (51)$$



9-rasm. Magnit qayta ulanish jarayonining quvvati tez aylanadigan Kerr-Newman-MOG qora o‘rasi uchun chizilgan. Yuqori qator, chap: P_{MR}/w_0 o‘zgarmas $a, Q,$ va ξ lar uchun plazma magnitlanishi σ_0 ning turli kombinatsiyalari uchun chizilgan. Yuqori qator, o‘ng: P_{MR}/w_0 o‘zgarmas a, Q , and σ_0 lar uchun orientatsiya burchagi ξ ning turli kombinatsiyalari uchun chizilgan. o‘rta qator yuqori qatorda kuzatilgani bilan bir xil, $P_{MR}/w_0\alpha$ MOG parametrining mavjudligi natijasida chizilgan. Quyi qator: P_{MR}/w_0 Kerr va Kerr-NewmanMOG qora o‘ra holatlari uchun chizilgan, shu bilan birga $Q = 0.1, \sigma_0 = 10^4$ va $\xi = \pi/20$ o‘zgarmagan holda saqlanadi. E'tibor bering, spin parametrining ekstremal qiymati $a_{ext} = (1 + \alpha - Q^2)^{1/2}$.

$\sigma_0 = B_0^2/w$ va ξ lar mos ravishda plazma magnitlanishi va magnit maydon hamda ekvator tekisligidagi chiqayotgan plazma yo‘nalishlari o‘rtasidagi orientatsiya burchagiga ishora qiladi. Magnit qayta ulanish jarayoni asosida qora o‘raning aylanish energiyasini olish uchun plazma tezlashtirilganda gidrodinamik energiya musbat bo‘ladi. Aksincha, energiya qora o‘ra gorizonti yaqinida sekinlashganda salbiy bo‘ladi, bu Penrose jarayoni uchun kuzatilganiga o‘xshaydi. Shunday qilib, magnit qayta ulash jarayonida olinadigan energiya musbat bo‘lishi kerak va har doim issiqlik energiyasidan va plazmaning tinclikdagi massasidan ancha katta bo‘lishi kerak. Bundan tashqari, plazmani holat tenglamasi shartini qanoatlantiradigan relativistik issiq plazma deb faraz qilamiz, ya’ni $w = 4p$. Shu nuqtai nazardan, plazmaning cheksizda o‘lchanadigan tezlashtirilgan va sekinlashtirilgan energiyalari quyidagicha yoziladi:

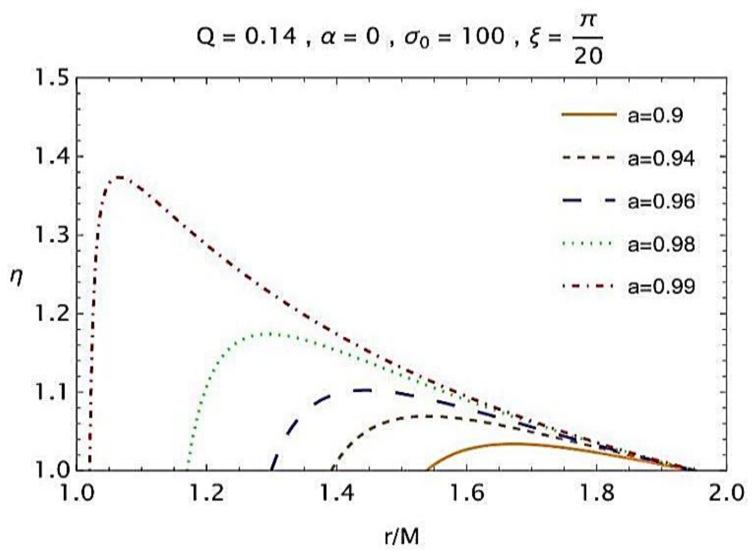
$$\epsilon_-^\infty < 0 \text{ and } \Delta\epsilon_+^\infty > 0 \tag{52}$$

bu yerda $\Delta\epsilon_+^\infty$ quyidagicha berilgan

$$\Delta\epsilon_+^\infty = \epsilon_+ - \left(1 - \frac{\Gamma}{\Gamma - 1}\frac{p}{w}\right) > 0 \tag{53}$$

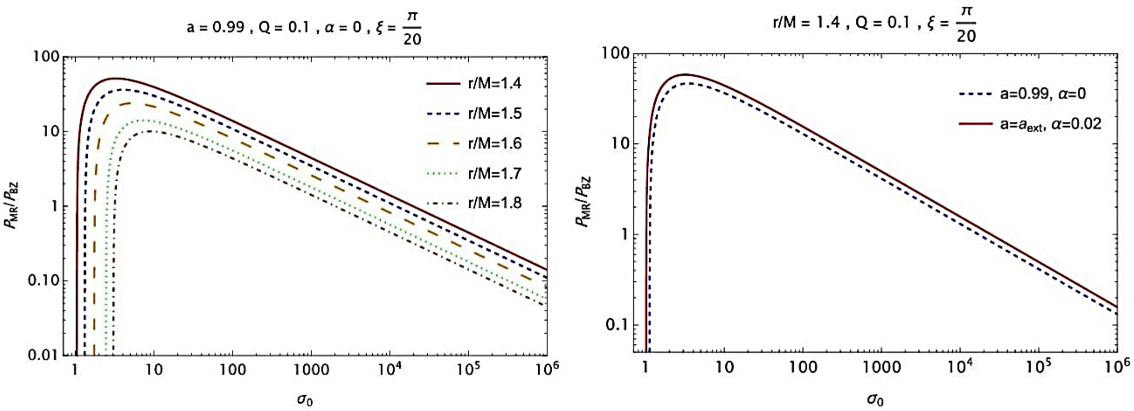
Politropik indeksni $\Gamma = 4/3$ sifatida qabul qilish relyativistik issiq plazma uchun $\Delta\epsilon_+^\infty = \epsilon_+^\infty > 0$ shaklida bo‘lishiga imkon beradi.

Endi biz plazmaning tezlashtirilgan ϵ_+^∞ va sekinlashtirilgan ϵ_-^∞ energiyalarini tahlil qilamiz va Kerr-Newman-MOG qora o‘rasi uchun magnit qayta ulanish jarayoni orqali energiya olishning ajoyib jihatlarini o‘rganamiz. Biroq, ϵ_+^∞ va ϵ_-^∞ energiyalarining analitik shakllari aniq ko‘rsatish uchun juda uzun va murakkab ifodalar bo‘lib chiqadi. Shunday qilib, Fig. 7 da biz turli mumkin bo‘lgan holatlar uchun plazma magnitlanishining funktsiyasi sifatida ϵ_+^∞ va ϵ_-^∞ plazma energiyasini ko‘rsatamiz. Figure 8 magnitlanish parametri σ_0 (yuqori qator, chap va o‘ng panellar) va orientatsiya burchagi ξ ning (pastki qator, chap va o‘ng panellar) faza fazosining hududlari $(a, r/M)$ da rolini aks ettiradi.



10-rasm. Magnit qayta ulanish samaradorligining r/M ga bog‘lanishi spin parametr (chap) va MOG parametr α (o‘ng) larning turli kombinatsiyalari uchun qora o‘ra zaryadi Q o‘zgarmas bo‘lgan holda. Bu yerda biz plazma magnitlanishi uchun $\sigma_0 = 100$ ni o‘rnatdik $\xi = \pi/20$ qayta ulanadigan magnit ega bo‘lishi mumkin bo‘lgan yo‘nalish burchagi uchun.

Bu yerda ta'kidlanishi kerak bo'lgan eng muhim nuqta shundaki, Fig. 8 ning yuqori va pastki o'ng panellarida ko'rsatilganidek, qora o'ra zaryadi va MOG parametrining birgalikdagi ta'siri energiya olish sharti uchun faza maydonini kengaytirishi mumkin, ya'ni $\delta\epsilon_+^\infty > 0$ (kulrang maydon). Keyinchalik, Kerr-Newman-MOG qora o'ralari uchun magnit qayta ulanish jarayoni orqali quvvat va energiya samaradorligini o'rganamiz.



11-rasm. Quvvatning nisbati P_{MR}/P_{BZ} plazma magnitlanishi σ_0 ga bog'lanishi. Chap: P_{MR}/P_{BZ} qora o'ra parametrlari va orientatsiya burchagi o'zgarmas bo'lgan holda r/M ning turli kombinatsiyalari uchun chizilgan. o'ng: P_{MR}/P_{BZ} MOG parametrning ikkita qiymatlari va unga mos keluvchi spin parametr uchun chizilgan.

Quvvat va magnit qayta ulanish samaradorligi

Ushbu tadqiqotda biz magnit qayta ulanish jarayoni orqali quvvat va energiya samaradorligini ko'rib chiqamiz va Kerr-Newman-MOG qora o'ralari uchun bu miqdorlarni raqamli baholashga murojaat qilamiz. Shuni ta'kidlash kerakki, energiya samaradorligi va quvvat birlik vaqt ichida qora o'ra tomonidan so'rilgan sekinlashtirilgan plazmaning manfiy energiyasi bilan kuchli bog'liqdir. Biz birinchi navbatda energiya qazib olish kuchiga murojaat qilamiz, unda qora o'radan chiqadigan plazma tufayli olinadigan quvvatni quyidagicha aniqlash mumkin

$$P_{MR} = -\epsilon_-^\infty w_0 A_{\rm in}\, U_{in} \tag{54}$$

bu yerda $U_{\rm in}$ rejimni belgilaydi, ya'ni $U_{\rm in} = \mathcal{O}(10^{-1})$ to'qnashuvsiz rejimni, $U_{\rm in} = \mathcal{O}(10^{-2})$ to'qnashuv rejimini bildiradi. Bundan tashqari, yuqoridagi ifodada berilgan $A_{\rm in}$ oqim uchun kesma maydonini belgilaydi va aylanuvchi qora o'ralar uchun $A_{in} = \left(r_E^2 - r_{ph}^2\right)$ sifatida baholanadi.

Keling, magnit qayta ulanish orqali energiya olishning yana bir muhim jihatiga murojaat qilaylik. Magnit qayta ulanish qanchalik samarali model ekanligini tushunish uchun bu yerda ko'rib chiqilgan model tomonidan olinishi mumkin bo'lgan energiya chiqishini tahlil qilish kerak. Shuning uchun biz Kerr-Newman-MOG qora o'ra tomonidan chiqarilgan energiyaning umumiy miqdorini baholashimiz kerak. Avval shuni ta'kidlashni istardikki, magnit qayta ulanish paytida magnit maydon energiyasi taqsimlangandan so'ng energiyaning ikki qismidan, ya'ni sekinlashtirilgan plazma va tezlashtirilgan plazma energiyasidan iborat. Manfiy energiyaga ega bo'lgan birinchisi qora o'ra tomonidan so'riladi, ikkinchisi esa mustaqil ravishda yuqori musbat energiyaga ega bo'lsa, qora o'radan kattaroq r masofalarga chiqadi. Shu nuqtai nazardan, magnit qayta ulanish orqali plazmaning energiya samaradorligi odatda quyidagicha aniqlanishi mumkin

$$\eta = \frac{\epsilon_+^\infty}{\epsilon_+^\infty + \epsilon_-^\infty} \tag{55}$$

bu yerda ϵ_+^∞ va ϵ_-^∞ lar yuqorida aytib o'tilganidek mos ravishda tezlashtirilgan va sekinlashtirilgan plazmani aniqlaydi. Bu yerda ta'kidlash kerak bo'lgan asosiy nuqta shundaki, plazma energiyasi qora o'radan chiqishi uchun $\eta = \epsilon_+^\infty/(\epsilon_+^\infty + \epsilon_-^\infty) > 1$ sharti mavjud energiya samaradorligi uchun har doim qanoatlantirilishi kerak.

Nihoyat, tezkor magnit qayta ulanishda energiya ajratib olish tezligini baholash uchun magnit qayta ulanish va Blandford-Znajek mexanizmlarining quvvatini solishtirishga o'tamiz. Buning uchun biz birinchi navbatda gorizontda BZ mexanizmi uchun energiya olish tezligini ko'rib chiqamiz va u quyidagicha aniqlanadi

$$P_{\rm BZ} = \frac{\kappa}{16\pi}\Phi_{BH}^2\Omega_H^2[1 + \chi_1\Omega_H^2 + \chi_2\Omega_H^4 + \mathcal{O}(\Omega_H^6)] \tag{56}$$

bu yerda Φ_{BH} va Ω_H mos ravishda magnit oqimini va quyidagicha berilgan gorizontdagi Ω_H burchak tezligini bildiradi

$$\Omega_H = \frac{a}{2\mathcal{M}r - Q^2 - \frac{\alpha}{1+\alpha}\mathcal{M}^2} \tag{57}$$

κ, χ_1 va χ_2 esa raqamli o'zgarmaslarga ishora qiladi. Bu yerda biz κ ning geometrik konfiguratsiya bilan aloqasi borligini ta'kidlaymiz. Keyin magnit oqim $\Phi_{BH} = \frac{1}{2}\int_\theta\int_\phi |B^r| dA_{\theta\phi}$ orqali aniqlanadi va qo'shimcha tahlil, soddalik uchun $\Phi_{BH} \sim 2\pi B_0 r_H^2 \sin\xi$ deb taxmin qilamiz. Bu yerda shuni ta'kidlaymizki, magnit maydon konfiguratsiyasi uchun orientatsiya burchagi ξ faqat past kengliklarda yaxshi baho sifatida qaralishi mumkin. Darhaqiqat, barcha kengliklarda magnit maydon konfiguratsiyasi bilan Φ_{BH} magnit oqimini baholash kerak. P_{MR}/P_{BZ} bu ikki quvvat nisbatini jamlagan holda quyidagicha yozish mumkin

$$\frac{P_{MR}}{P_{BZ}} = \frac{-4\epsilon_-^\infty w_0 A_{in} U_{in}}{\pi\kappa\Omega_H^2 r_H^4 \sigma_0 \sin^2\xi(1 + \chi_1\Omega_H^2 + \chi_2\Omega_H^4)} \tag{58}$$

Bu yerda biz ushbu raqamli konstantalarning qiymatlari $\chi_1 \approx 1.38$ va $\chi_2 \approx -9.2$, magnit maydon geometriyasi tufayli esa $\kappa \approx 0.044$ deb taxmin qilinganligini ta'kidlaymiz.

Dissertatsiyaning uchinchi "**Elektrik Penrose jarayoni va 4D zaryadlangan Eynshteyn-Gauss-Bonnet qora o'rasi atrofidagi akkretsiya diski**" bobida, yana bir muqobil model hisoblangan elektrik Penrose jarayoni o‘rganilgan. Zaryadlangan qora o‘radan zaryadli zarralar orqali energiya ajratib olish tahlil qilingan.

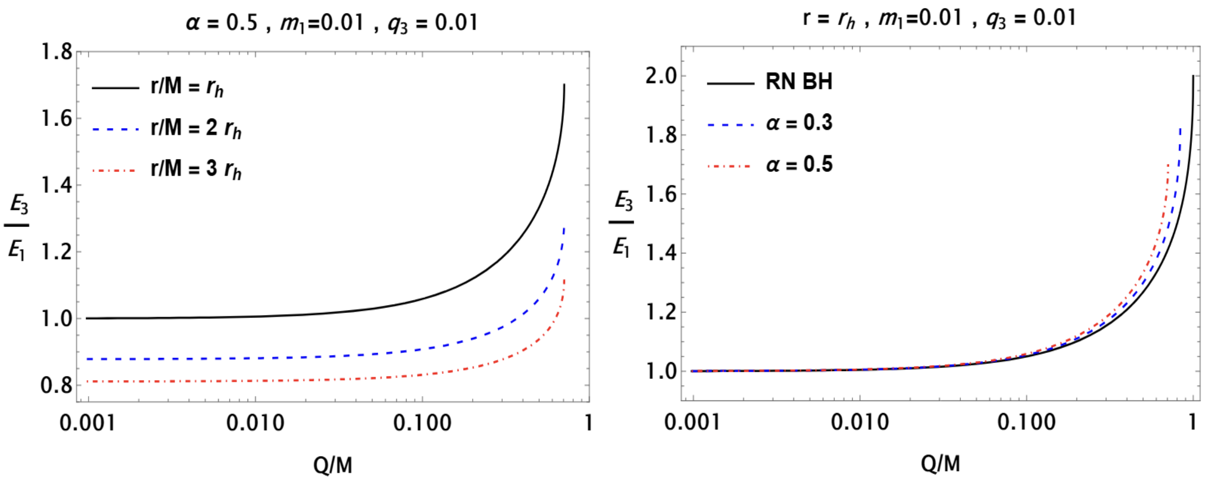
4D EGB nazariyasida qora o‘raning chiziqli elementi quyidagicha yoziladi

$$\boldsymbol{ds^2 = -\mathrm{f}(r)dt^2 + \frac{1}{\mathrm{f}(r)}dr^2 + r^2 d\Omega^2} \qquad \textbf{(59)}$$

bu yerda metrik funksiya quyidagicha

$$\mathrm{f}(r) = 1 + \frac{\mathrm{r}^2}{2\alpha}\left(1 - \sqrt{1 + \frac{4\alpha}{\mathrm{r}^2}\left(\frac{2\mathrm{M}}{\mathrm{r}} - \frac{\mathrm{Q}^2}{\mathrm{r}^2}\right)}\right)$$

berilgan. M umumiy massa va Q qora o‘raning elektr zaryadi. Ushbu fazo-vaqt atrofidagi zarralar uchun saqlanish qonunlarini yozib, energiya samaradorlik ifodasini keltirib chiqaramiz hamda tegishli grafiklarda ifodalaymiz.



12-rasm. Ionlashgan va neytral zarralar energiyalarining nisbati (elektrik Penrose jarayonining energiya samaradorligi) qora o‘ra zaryadi funksiyasi sifatida. Chap panel: energiya samaradorligi ionizatsiya nuqtalari $\mathbf{r/M}$ ning turli qiymatlari uchun chizilgan. O‘ng panel: energiya samaradorligi α parametrning turli qiymatlari uchun chizilgan.

Ta’kidlash joizki, elektrik Penrose jarayoning samaradorligi oddiy Penrose jarayoni samaradorligidan kattaligi dissertatsiya ishida ko‘rsatilgan.

XULOSA

“Qora o‘ralar mimikerlarining kuzatuv xossalari va ularning energetikasiga muqobil modellar”mavzusida olib borilgan tadqiqotlar asosida quyidagi xulosalar keltirildi:

1. MOG parametri va qora o‘ra zaryadining qo‘shma ta’siri natijasida entalpiya uchun tezlashtirilgan va sekinlashtirilgan energiya o‘sib borishi ko‘rsatildi, bu esa magnit qayta ulanish orqali olinishi mumkin bo‘lgan mustaqil yuqori energiyaga olib keladi. Bundan tashqari qora o‘ra zaryadi va MOG parametrining qo‘shma ta’siri energiya ajratib olish holati uchun fazalar maydonini kengaytirishi mumkinligi

aniqlandi, bu esa Kerr-Newman-MOG qora oʻrasidan magnit qayta ulanish orqali yuqori energiya ajratib olishga olib keladi.

2. Qora oʻra zaryadi va MOG parametrining qoʻshma ta’siri Kerr qora oʻrasiga nisbatan magnit qayta ulanish orqali yuqori quvvatga olib kelishi koʻrsatilgan. Shuningdek, MOG parametri va qora oʻra zaryadining qoʻshma ta'siri yuqori energiya samaradorligiga yaqinlashishda muhim rol oʻynashi aniqlandi: tortishish MOG parametri mavjudligi sababli energiya olishning samaradorligi maksimal mumkin boʻlgan maksimal qiymatga yetadi.

3. Magnit qayta ulanish va Blandford-Znajek mexanizmi tufayli samaradorlik nisbati qora oʻra zaryadi va MOG parametrining ta’siri tufayli ortishi koʻrsatilgan. Magnit qayta ulanish Blandford-Znajek mexanizmiga qaraganda ancha samarali ekanligi ta’kidlandi: natijada samaradorlik Kerr qora oʻrasiga nisbatan oshadi. MOG parametri ramkani tortish effekti tufayli magnit maydon chiziqlarining qayta konfiguratsiyasiga kuchli ta’sir qiladigan yanada tez aylanishga olib kelishi mumkin degan xulosaga kelindi.

4. Neytrino tebranishlarining ehtimolligi geometriyaning Shvartsshilddan chetga chiqishini tavsiflovchi Rezzolla-Zhidenko metrikasidagi parametrlarning qiymatlariga bog'liqligi koʻrsatilgan. Neytrinolarni kuzatishlar natijasida geometriyaning xususiyatlarini aniqlash mumkinligi aniqlandi.

5. Ma’lum uzunlikdan keyin neytrinolarning kogerentligini yoʻqotishi va dekogerentlik uzunligi mutlaq neytrino massalari bilan solishtirganda Rezzolla-Zhidenko metrikasining deformatsiya parametrlaridan ahamiyatsiz ta’sir qilishi koʻrsatilgan.

SCIENTIFIC COUNCIL DSc.03/31.03.2022.T/FM.10.04 ON AWARD OF SCIENTIFIC DEGREE AT INSTITUTE OF FUNDAMENTAL AND APPLIED RESEARCH "TIIAME" NATIONAL RESEARCH UNIVERSITY

INSTITUTE OF FUNDAMENTAL AND APPLIED RESEARCH

ALLOKULOV MIRZABEK ULUGBEK UGLI

OBSERVATIONAL PROPERTIES OF BLACK HOLE MIMICKERS AND ALTERNATIVE MODELS TO THEIR ENERGETICS

01.03.01 – Astronomy
01.04.02 – Theoretical Physics

DOCTOR OF PHILOSOPHY IN PHYSICAL AND MATHEMATICAL SCIENCES (PhD)
ABSTRACT OF THE DISSERTATION

Tashkent – 2024

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Scientific supervisors:	**Ahmedov Bobomurat Juraevich** Doctor of physical and mathematical sciences, academician
	Shaymatov Sanjar Ruzimurotovich Doctor of physical and mathematical sciences
	Daniele Malafarina Doctor of physical and mathematical sciences, professor
Official opponents:	**Abishev Medeu Erzhanovich** Doctor of physical and mathematical sciences, academician
	Kholikov Shukirjon Sodikovich Doctor of physical and mathematical sciences
Leading organization:	**Samarkand State University named after Sharof Rashidov**

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The dissertation can be looked through at the Information Resource Center of the Institute of Fundamental and Applied Research under the National Research University "TIIAME" (registered under №_). (Address: 100000, Tashkent city, 39 Qori Niyaziy str.,Institute of Fundamental and Applied Research, hall 205; ph.: 71 237-09-61

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B.A. Toshmatov
Vice Chairman of the Scientific Council on Award of Scientific Degrees, DSc.

Sh.N. Mardonov
Scientific Secretary of Scientific Council on Award of Scientific Degrees, DSc.

B.M. Narzilloev
Chairman of the Scientific Seminar of the Scientific Council on Award of Scientific Degrees, DSc.

INTRODUCTION (presentation abstract)

Relevance and necessity of the topic. Nowadays, modern astronomical observations from the LIGO-VIRGO and Event Horizon Telescope allow for the direct detection of gravitational waves from the merger of black holes and binary neutron stars in close binary systems, as well as the imaging of the supermassive black hole at the center of the M87 galaxy. The direct detection of gravitational waves, along with the observation and analysis of the supermassive black hole image through these instruments, provides explanations for astrophysical phenomena in a strong gravitational field regime. These modern observations are crucial for finding new generalized solutions in alternative theories of gravity and for investigating issues related to the precise measurement of parameters and the nature of dark matter. Despite the astronomical observations mentioned above, questions remain regarding the nature of the formation of high-energy astrophysical phenomena involving rotating black holes and the accurate determination and the precise measurement of parameters associated with the geometry of astrophysical black hole candidates. Additionally, while Einstein's general theory of relativity (GR) is a significant and main theory, it does not fully address the unique singularity and dark matter problems of black holes. Therefore, the development of promising alternative theories that offer a fundamental understanding of these issues, as well as obtaining precise constraints on their parameters through the creation of new models and alternative generalized space-time geometries, necessitates testing.

It is important to note that in recent years, our country has been increasingly focused on advancing both fundamental and applied research areas. The advancement of theoretical astrophysical research, a promising field, is particularly important today. The key areas of fundamental research and development, along with their practical applications for advancing science in our country, are outlined in the Strategy[1] for the Further Development of the Republic of Uzbekistan for 2022–2026. The study of the energetics of astrophysical objects remains one of the important issues in the field of fundamental research.

This research aligns with the objectives outlined in the following state regulatory documents: Presidential Decree No. PD-4947, "On the Strategy of Actions for the Further Development of the Republic of Uzbekistan," dated February 7, 2017; Presidential Resolution No. PR-2789, "On Measures for Further Improvement of the Activities of the Academy of Sciences, Organization, Management, and Financing of Research Activities," dated February 18, 2017; among others.

Conformity of the research to the main priorities of science and technology development of the Republic. The research has been conducted in line with the priority areas of science and technology in the Republic of Uzbekistan, specifically under the category II: "Power, Energy, and Resource-Saving."

Connection of the topic of the research topic to the scientific works ofhigher education and research institutions, where the dissertation is carriedout. The

[1] Decree No. PF-60 of the President of the Republic of Uzbekistan dated January 1, 2022 “On the Development Strategy of New Uzbekistan for 2022-2026”

presentation was completed as part of scientific projects funded by the Agency of Innovative Development, specifically project F-FA-2021-510, "Investigations of Nuclear Matter of Neutron Stars in Modified Gravity."

The aim of the research is to study the observational properties of black hole mimickers and find alternative models for their energetics.

The tasks of the research:

to analyze the impact of the rotating Kerr- Newman-MOG black hole on the magnetic reconnection via Comisso-Asenjo mechanism;

to study the energy extraction by the magnetic reconnection process that occurs continuously inside the ergosphere due to black hole spin;

to explore the energy efficiency of energy extraction and the power as a function of plasma magnetization, magnetic field orientation, and black hole spin and MOG parameters by imposing all required conditions;

to explore the accelerated and decelerated energies of the plasma for the energy extraction through the magnetic reconnection;

to study the phase-space region for satisfying the required condition for energy extraction;

to study the power and the energy efficiency of a rapidly rotating Kerr-Newman-MOG black hole in order to understand how the magnetic reconnection is efficient as compared to the Kerr black hole case;

to make an estimation of the rate of energy extraction under the fast magnetic reconnection by comparing the power of the magnetic reconnection and Blandford-Znajek mechanisms;

to analyze the behavior of efficiency rate against the plasma magnetization for various possible cases;

to study the oscillations of neutrino flavors propagating in the Rezzolla-Zhidenko spacetime and determined how the detection of neutrinos may in principle be used to constrain the geometry in which they travelled.

The object of the research are astrophysical compact objects, particle dynamics, alternative models for the energetics of astrophysical compact objects.

The subject of the research are observational properties of black hole mimickers, alternative models for their energetics, particle dynamics around astrophysical compact objects, analytical and numerical methods for solving differential equations of the motion of particles.

The methods of the research are methods of theoretical physics and astrophysics, modern methods of theoretical astrophysics and mathematical physics, analytical and numerical methods for solving differential equations related to field and particle dynamics.

The scientific novelty of the research is the following:

The energy efficiency and power of the energy extraction have been studied by imposing all necessary conditions as a function of plasma magnetization, magnetic field direction, and black hole spin and MOG parameters.

For the first time the influence of the rotating Kerr-Newman-MOG black hole on the magnetic reconnection through the Comisso-Asenjo mechanism has been analyzed.

For the first time the power and energy efficiency of fast rotating Kerr-Newman-MOG black holes have been investigated to understand how efficient magnetic reconnection is compared to the Kerr black hole case.

By comparing the power of magnetic reconnection and Blandford-Znajek mechanisms, the rate of energy extraction in fast magnetic reconnection has been evaluated.

For the first time the oscillations of neutrino flavors propagating in the Rezzolla-Zhidenko spacetime is studied and determined how the detection of neutrinos may in principle be used to constrain the geometry in which they travelled.

The practical results of the research are the following:

The influence of the rotating Kerr-Newman-MOG black hole on the magnetic reconnection through the Comisso-Asenjo mechanism has been analyzed.

The energy efficiency and power of the energy extraction have been studied by imposing all necessary conditions as a function of plasma magnetization, magnetic field direction, and black hole spin and MOG parameters.

The power and energy efficiency of fast rotating Kerr-Newman-MOG black holes have been investigated to understand how efficient magnetic reconnection is compared to the Kerr black hole case.

By comparing the power of magnetic reconnection and Blandford-Znajek mechanisms, the rate of energy extraction in fast magnetic reconnection has been evaluated.

It has been studied the oscillations of neutrino flavors propagating in the Rezzolla-Zhidenko spacetime and determined how the detection of neutrinos may in principle be used to constrain the geometry in which they travelled.

The reliability of the research results is ensured through the application of established modern methods in mathematical physics, computational mathematics, and relativistic astrophysics. The results were derived strictly within the mathematical framework of general relativity and theoretical physics. Modern numerical and analytical calculation methods are also applied, with results compared to existing observational data and findings from other researchers. The structured conclusions of the presentation align with the fundamental principles of astrophysics concerning compact objects.

The scientific and practical significance of the research results. The scientific significance of the research results is that the selected Rezzola-Zhidenko metric includes all spherically symmetric static metrics.

The practical significance of the research results is that the proposed model for black hole mimickers can play an important role in explaining high-energy processes.

Implementation of the research results. Based on the energetics of astrophysically compact objects:

scientific results obtained on the energetics of astrophysically compact objects have been used in the works of foreign researchers, in foreign journals with a high impact factor (Physical Review D, Volume 110, article id. 063003 Web-Sc, IF: 5.407 va Physical Review D, Volume 109, article id. 084066 Web-Sc, IF: 5.407).

Approbation of the research results. The research results were reported and discussed at 2 international and 1 local scientific conferences.

Publication of research results. The results of PhD research have been presented in 14 peer-reviewed articles published in scientific journals recommended by Supreme Attestation Commission at the Ministry of higer education, science and innovations of the Republic of Uzbekistan.

Volume and structure of the dissertation. The dissertation consists of an introduction, three chapters, conclusion and a bibliography. The size of the dissertation is 120 pages.

MAIN CONTENT OF THE DISSERTATION

The Introduction of the dissertation indicates the relevance and necessity of the topic, the correspondence of the study to the priority areas of development of science and technology of the republic, the degree of study of the problem, its connection with the research plans of the higher educational institution in which the dissertation was completed, and the goal, objectives, object of study, brief information about the subject, methods, scientific novelty, practical result, reliability, scientific and practical significance of the results, putting the results into practice, approval of the results, publication of the results, as well as the structure and scope of the dissertation.

The first chapter entitled "**Gravitational lensing of neutrinos in the Rezzolla-Zhidenko spacetime**" a brief overview about the topic, the purpose of the dissertation and problems on black holes and black hole mimickers have been provided. Moreover, the probability of neutrino oscillations have been explored in the given spacetime.

The line element of a general spherically symmetric static metric in a spherical coordinates $\{t, r, \theta, \phi\}$ can be written as

$$ds^2 = -N^2(r)dt^2 + \frac{B^2(r)}{N^2(r)}dr^2 + r^2 d\Omega^2 \tag{1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the usual spherical part of the metric, and N and B are functions of the radial coordinate r only. The radial location of the event horizon is marked as $r = r_0 > 0$ and this definition implies that

$$N^2(r_0) = 0 \tag{2}$$

We neglect any cosmological effect, so that the line element (1) can be taken as asymptotically flat. Then the radial coordinate may be compactified by introducing a dimensionless variable x, given by

$$x = 1 - \frac{r_0}{r} \tag{3}$$

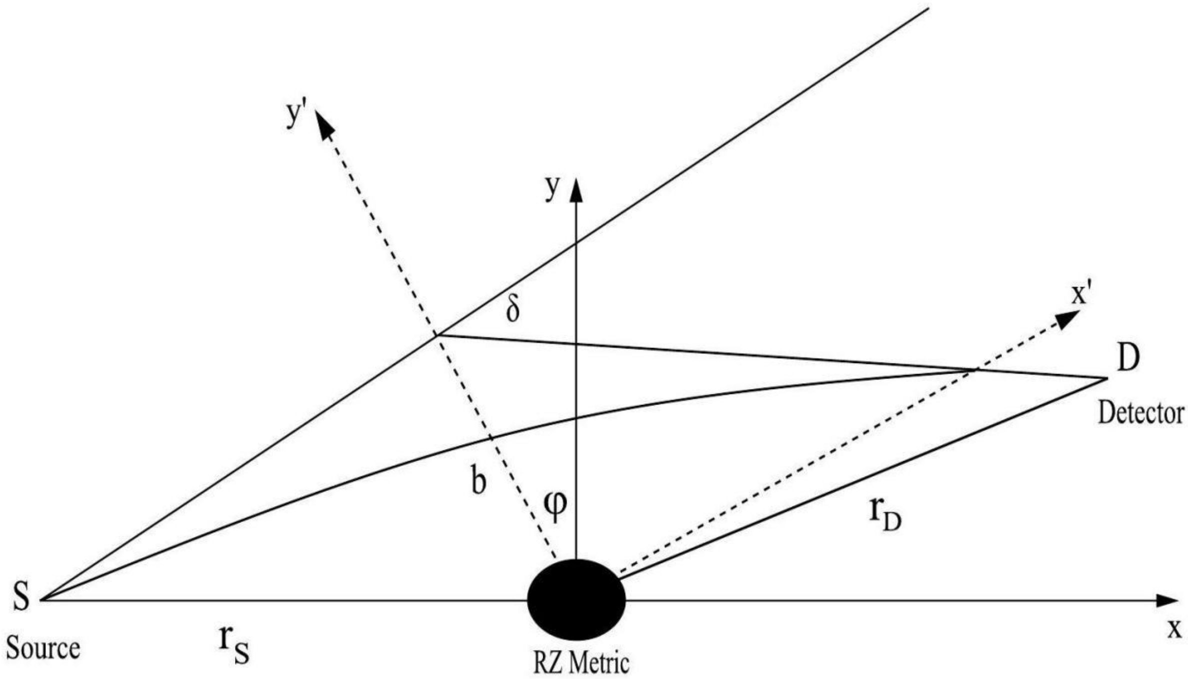

Figure 1. Schematic diagram of weak lensing of neutrinos in the RZ spacetime. Neutrinos propagate from the source *S* to detector *D* in the exterior of a static and spherical massive object described by the RZ spacetime.

Evidently, $x = 0$ corresponds to the location of the event horizon and $x = 1$ corresponds to spatial infinity. The metric function $N(r)$ can be rewritten in terms of the dimensionless variable x as

$$N^2(x) = xA(x) \tag{4}$$

with

$$A(x) > 0 \text{ for } 0 \le x \le 1 \tag{5}$$

Now the functions A and B can be rewritten by introducing dimensionless parameters ϵ, a_0, and b_0 as

$$A(x) = 1 - \epsilon(1-x) + a_0(1-x)^2 + \tilde{A}(x)(1-x)^3,\ B(x) = 1 + b_0(1-x) + \tilde{B}(x)(1-x)^2 \tag{6}$$

where the functions $\tilde{A}(x)$ and $\tilde{B}(x)$ are used to describe the asymptotic behavior of the metric.

One can express the functions $\tilde{A}(x)$ and $\tilde{B}(x)$ in terms of continued fractions

$$\tilde{A}(x) = \cfrac{a_1}{1 + \cfrac{a_2 x}{1 + \cfrac{a_3 x}{1 + \cdots}}},\ \tilde{B}(x) = \cfrac{b_1}{1 + \cfrac{b_2 x}{1 + \cfrac{b_3 x}{1 + \cdots}}} \tag{7}$$

where a_1, a_2, a_3 ... and b_1, b_2, b_3 ... are dimensionless constants. Then the metric functions in (1) can be expanded up to any desired order. We obtain

$$N^2(x) = 1 - (1+\epsilon)(1-x) + a_0(1-x)^2 + (a_1 - a_0 + \epsilon)(1-x)^3 - a_1(1-x)^4$$
$$B^2(x) = (1 + b_0(1-x) + b_1(1-x)^2)^2 \quad (8)$$

Notice here that the parameter ϵ describes the departure of the event horizon radius r_0 from $2M$ as it is related to the event horizon by

$$\epsilon = -\left(1 - \frac{2M}{r_0}\right) \quad (9)$$

where M is the ADM mass of the spacetime. In order to retrieve the Schwarzschild spacetime for which the horizon is at $r_0 = 2M$ we must have $\epsilon = 0$ and $a_i = b_i = 0 (i = 0,1,2 \dots)$. In general, the functions N and B can be expressed in terms of the parameterized post Newtonian (PPN) parameters as

$$N^2 = 1 - \frac{2M}{r} + (\beta - \gamma)\frac{2M^2}{r^2} + \mathcal{O}(1/r^3), \; \frac{B^2}{N^2} = 1 + \gamma\frac{2M}{r} + \mathcal{O}(1/r^2) \quad (10)$$

so that we get

$$\beta - \gamma = \frac{2a_0}{(1+\epsilon)^2}, \; \gamma - 1 = \frac{2b_0}{1+\epsilon} \quad (11)$$

The parametrization then allows to describe deviations from the Schwarzschild metric at every perturbative order. In the following we shall return to spherical coordinates and write the line element Eq. (1) in the following form

$$ds^2 = -\mathcal{A}dt^2 + \mathcal{B}dr^2 + \mathcal{C}d\theta^2 + \mathcal{D}d\phi^2 \quad (12)$$

where $\mathcal{A}$ and $\mathcal{B}$ are defined from A and B by keeping only first order terms, namely $\epsilon \neq 0$ and $b_0 \neq 0$ with all the other parameters vanishing. This gives

$$\mathcal{A}(r) = N^2(r) = 1 - (1+\epsilon)\left(\frac{r_0}{r}\right), \quad \mathcal{B}(r) = \frac{B^2(r)}{N^2(r)} = \frac{\left(1 + b_0\left(\frac{r_0}{r}\right)\right)^2}{1 - (1+\epsilon)\left(\frac{r_0}{r}\right)} \quad (13)$$
$$\mathcal{C}(r) = r^2, \qquad \mathcal{D}(r) = r^2 \sin^2\theta$$

Neutrino oscillations in flat spacetime

In weak interactions, neutrinos are produced and detected in different flavor eigenstates denoted by $|\nu_\alpha\rangle$ with $\alpha = e, \mu, \tau$, and the flavor eigenstates are superposition of mass eigenstates denoted by ν_i where $i = 1,2,3$. One can write the relations between mass and flavor eigenstates

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle \quad (14)$$

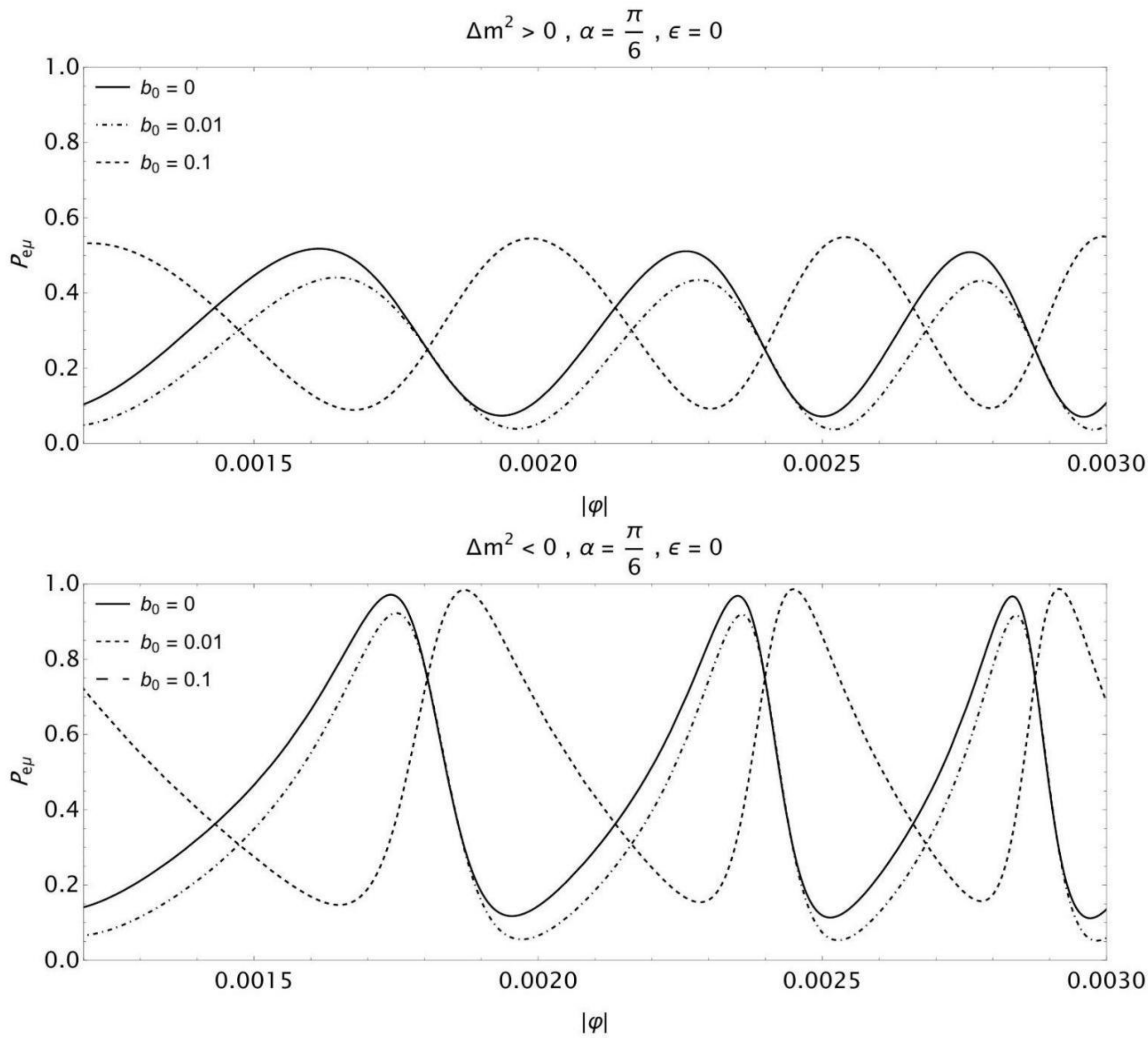

Figure 2. Top panel: neutrino oscillation probability as a function of azimuthal angle φ for $b_0 = 0$ (solid line), $b_0 = 0.01$ (dotted-dashed line) and $b_0 = 0.1$ (dashed line) for normal hierarchy $\Delta m^2 > 0$. Bottom panel: neutrino oscillation probability for $b_0 = 0$ (solid line), $b_0 = 0.01$ (dotted-dashed line) and $b_0 = 0.1$ (dashed line) for inverted hierarchy $\Delta m^2 < 0$. The mixing angle here is $\alpha = \pi/6$. Values of the other parameters are as follows: $M = 1M_{\odot}$, $\Delta m^2 = 10^{-3}\mathrm{eV}^2$, and the lightest neutrino here is considered to be massless.

where U is the 3×3 unitary mixing matrix. For three flavor neutrino oscillation, this is known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) leptonic mixing matrix. We assume that the neutrino wave-function is a plane wave and it propagates from a source S located at a spacetime event (t_S, x_S) to a detector D located at a spacetime event (t_D, x_D). So the wave-function at the detector point is given by

$$|\nu_i(t_D, x_D)\rangle = \exp(-i\Phi_i)|\nu_i(t_S, x_S)\rangle \tag{15}$$

where Φ_i is the phase of oscillation. Neutrinos are expected to be produced initially in the flavour eigenstate $|\nu_\alpha\rangle$ at S and then travel to the detector D. In that case, the probability of the change in neutrino flavour from ν_α to ν_β at D is given by

$$\mathcal{P}_{\alpha\beta} = \left|\langle \nu_\beta \mid \nu_\alpha(t_D, x_D)\rangle\right|^2 = \sum_{i,j} U_{\beta i} U^*_{\beta j} U_{\alpha j} U^*_{\alpha i} \exp\left(-i\left(\Phi_i - \Phi_j\right)\right) \tag{16}$$

The change in flavor can occur if $\Phi_i \neq \Phi_j$. Different neutrino mass eigenstates develop different phases Φ_i because of differences in their mass and energy/momentum which ultimately gives rise to neutrino oscillation phenomena. In flat spacetime, the phase is given by

$$\Phi_i = E_i(t_D - t_S) - \mathbf{p}_i \cdot (\mathbf{x}_D - \mathbf{x}_S) \tag{17}$$

It is typically assumed that all the mass eigenstates in a flavor eigenstate initially produced at the source have equal momentum or energy. Either of these assumptions together with $(t_D - t_S) \simeq |\mathbf{x_D} - \mathbf{x_S}|$ for relativistic neutrinos ($E_i \gg m_i$) leads to

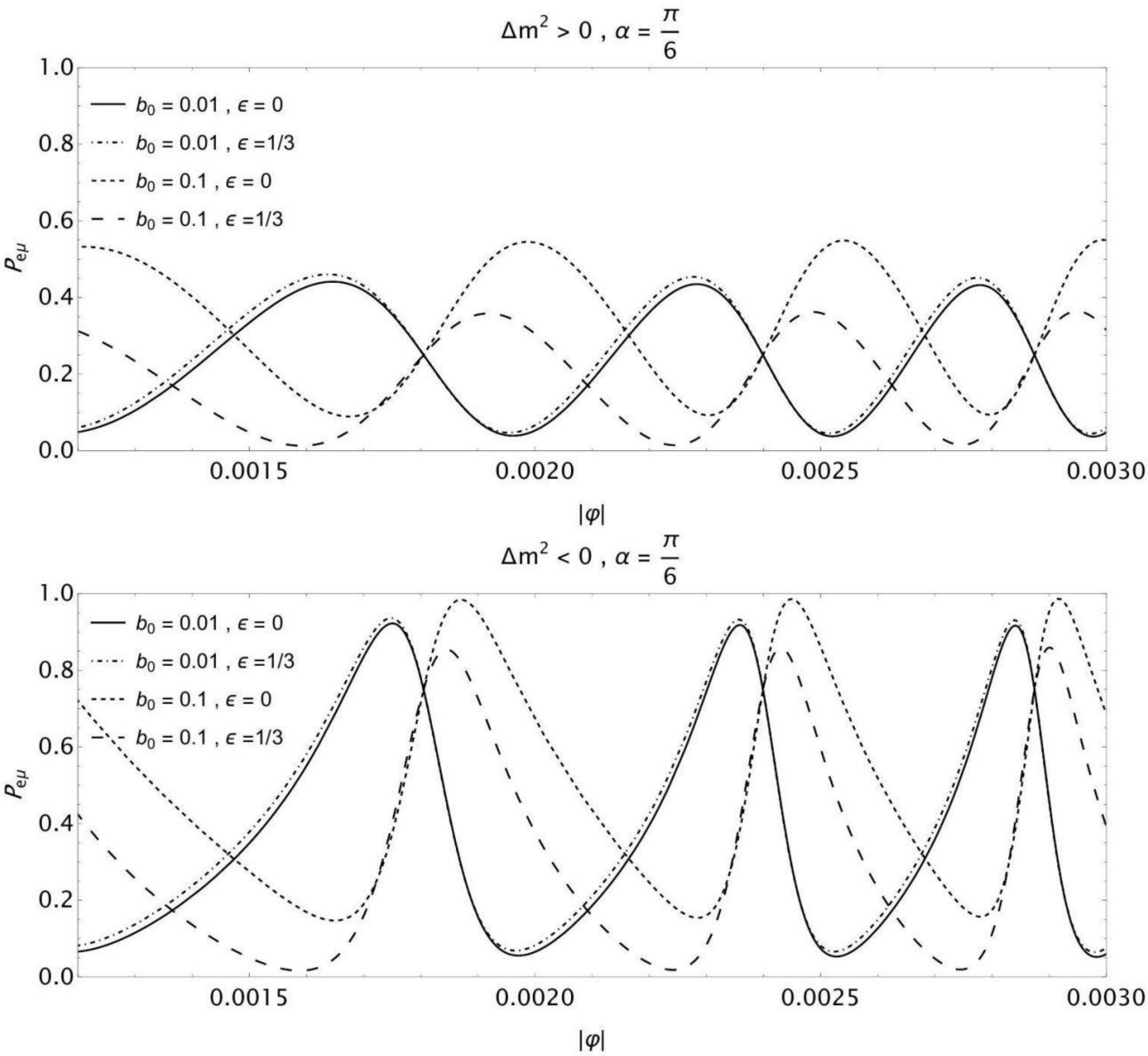

Figure 3. Top panel: neutrino oscillation probability as a function of azimuthal angle φ for $b_0 = 0.01, \epsilon = 0$ (solid line), $b_0 = 0.01, \epsilon = 1/3$ (dotted-dashed line), $b_0 = 0.1, \epsilon = 0$ (dashed line) and $b_0 = 0.1, \epsilon = 1/3$ (large dashed line) for normal hierarchy $\Delta m^2 > 0$. Bottom panel: neutrino oscillation probability for $b_0 = 0.01, \epsilon = 0$ (solid line), $b_0 = 0.01, \epsilon = 1/3$ (dotted-dashed line), $b_0 = 0.1, \epsilon = 0$ (dashed line) and $b_0 = 0.1, \epsilon = 1/3$ (large dashed line) for inverted hierarchy $\Delta m^2 < 0$. The mixing angle here is $\alpha = \pi/6$. Values of the other parameters are as follows: $M = 1M_{\odot}, \Delta m^2 = 10^{-3}\mathrm{eV}^2$.

$$\Delta\Phi_{ij} = \Phi_i - \Phi_j \simeq \frac{\Delta m_{ij}^2}{2E_0} |x_D - x_S| \tag{18}$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$, and E_0 is the average energy of the relativistic neutrinos produced at the source. To generalize the expression of the phase Φ_i for neutrino propagation in curved spacetime, Eq. (17) is written in the covariant form

$$\Phi_i = \int_S^D p_\mu^{(i)} dx^\mu \tag{19}$$

where

$$p_\mu^{(i)} = m_i g_{\mu\nu} \frac{dx^\nu}{ds} \tag{20}$$

is the canonical conjugate momentum to the coordinates x^ν and $g_{\mu\nu}$ and ds are the metric tensor and line element of the curved spacetime, respectively.

Neutrino oscillation probabilities

Having obtained the phase of oscillation for a gravitationally lensed neutrino, we would now like to calculate the oscillation probabilities of the same. We consider neutrinos with mass eigenstate ν_i travelling in the RZ spacetime. The main idea is that a neutrino emitted from the source S can travel along two different paths, say p and q, the proper distances of which are different and produce quantum interference at the detector D. A neutrino produced in a flavor eigenstate $|\nu_\alpha, S\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$ at the source S , evolves into

$$|\nu_\alpha, D\rangle = N \sum_i U_{\alpha i}^* \sum_p \exp(-i\Phi_i^p)|\nu_i\rangle \tag{21}$$

where N is a normalization constant and Φ_i^p with the impact parameter b_p that must be understood as dependent on the path p. The oscillation probability $\mathcal{P}$ for

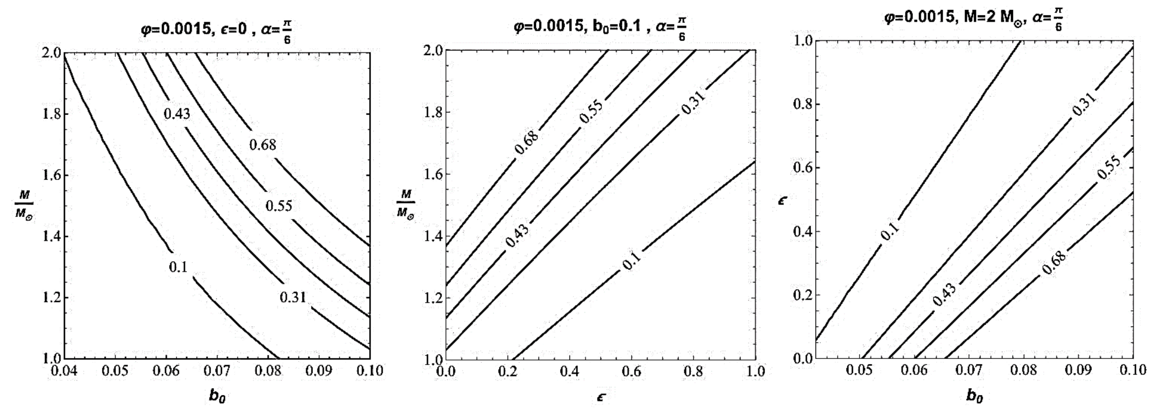

Figure 4. Left panel: The degeneracy between the determination of the mass parameter M and the RZ parameter b_0 with $\epsilon = 0$ for a given probability $\mathcal{P}_{e\mu}$ at a fixed angle ϕ is illustrated by the $2D$ contour plot of the implicit function $M(b_0)$ obtained from

$\mathcal{P}_{e\mu}(M, b_0) = \text{const}$. Middle panel: The degeneracy between the mass parameter M and the RZ parameter ϵ with $b_0 = 0.1$ for a given probability $\mathcal{P}_{e\mu}$ at a fixed angle ϕ is depicted by the $2D$ contour plot of the implicit function $M(\epsilon)$ derived from the equation $\mathcal{P}_{e\mu}(M, \epsilon) = \text{const}$. Right panel: The $2D$ contour plot of the implicit function $\epsilon(b_0)$ obtained from $\mathcal{P}_{e\mu}(\epsilon, b_0) = \text{const}$ illustrates the degeneracy between the parameter ϵ and the parameter b_0 of the RZ metric for a given probability $\mathcal{P}_{e\mu}$ at a fixed angle ϕ. Here, the mass parameter M is assumed to be $2M_\odot$. Each curve corresponds to a fixed value of $\mathcal{P}_{e\mu}$ as shown on the line itself.

neutrino flavor-changing from ν_α to ν_β at the detector can be written as

$$\mathcal{P}_{\alpha\beta} = \left|\langle \nu_\beta \mid \nu_\alpha, D\rangle\right|^2 = |N|^2 \sum_{i,j} U_{\beta i} U^*_{\beta j} U_{\alpha j} U^*_{\alpha i} \sum_{p,q} \exp\left(-i\Delta\Phi^{pq}_{ij}\right) \tag{22}$$

where

$$|N|^2 = \left(\sum_{i} |U_{\alpha i}|^2 \sum_{p,q} \exp\left(-i\Delta\Phi^{pq}_{ii}\right)\right)^{-1} \tag{23}$$

is the normalization factor and $\Delta\Phi^{pq}_{ij}$ is the phase difference given by

$$\Phi^{pq}_{ij} = \Phi^p_i - \Phi^q_j = \Delta m^2_{ij} A_{pq} + \Delta b^2_{pq} B_{ij} \tag{24}$$

where

$$A_{pq} = \frac{r_S + r_D}{2E_0}\left[1 + \frac{2M}{r_S + r_D} + \frac{2b_0 M}{(r_S + r_D)(1+\epsilon)} \ln\frac{r_S r_D}{M^2} - \frac{\Sigma b^2_{pq}}{4 r_S r_D}\right]$$

In the above equations, the quantities $\Delta m^2_{ij}, \Sigma m^2_{ij}, \Delta b^2_{pq}$ and Σb^2_{pq} are defined as follows

$$\Delta m^2_{ij} = m^2_i - m^2_j,\ \Sigma m^2_{ij} = m^2_i + m^2_j$$

In order to obtain a quantitative treatment of neutrino lensing in the RZ spacetime, we consider a simple toy model of two neutrino flavors, $\nu_e \to \nu_\mu$. For this transition, the oscillation probability becomes

$$\mathcal{P}^{\text{lens}}_{e\mu} = |N|^2 \sin^2 2\alpha \left[\sin^2\left(\Delta m^2 \frac{A_{11}}{2}\right) + +\sin^2\left(\Delta m^2 \frac{A_{22}}{2}\right) - \cos(\Delta b^2 B_{12})\cos(\Delta m^2 A_{12}) + \right.$$

$$\left. + \frac{1}{2}\cos(\Delta b^2 B_{11}) + \frac{1}{2}\cos(\Delta b^2 B_{22})\right] \tag{27}$$

and the normalization constant reduces to

$$|N|^2 = \frac{1}{2}\left(1 + \cos^2\alpha \cos(\Delta b^2 B_{11}) + \sin^2\alpha \cos(\Delta b^2 B_{22})\right)^{-1} \tag{28}$$

In this case $\Delta b^2 = \Delta b^2_{12}$ and $\Delta m^2 = \Delta m^2_{21}$. Notice that due to the function Δb^2

being even, $\mathcal{P}_{e\mu}^{\mathrm{lens}}$ does not change under the interchange of b_1 and b_2. However, the oscillation probability is sensitive to the neutrino mass ordering, leading to different results for $\Delta m^2 > 0$ and $\Delta m^2 < 0$. Additionally, we can see that the oscillation probability is sensitive to slight deviations from Schwarzschild spacetime and spherical symmetry through the involvement of the terms A_{pq} which depend on the RZ deformation parameters b_0 and ϵ.

Numerical results for the two flavor toy model

For a better understanding, we would like to see how the probability is affected by the variation of the lensing parameters in some realistic scenario. Therefore we need to express the impact parameter in terms of known geometrical quantities. To this aim we can refer to Fig.1. which illustrates a schematic diagram of weak lensing in the RZ spacetime. Neutrinos are produced at the point S and are lensed by a massive object whose geometry is described by the RZ spacetime. Eventually neutrinos are detected at the point D. From Fig 1 we see that we can express the distances from the massive object of the source and detector in a Cartesian plane $\{x, y\}$ as $r_S(x, y)$ and $r_D(x, y)$, respectively.

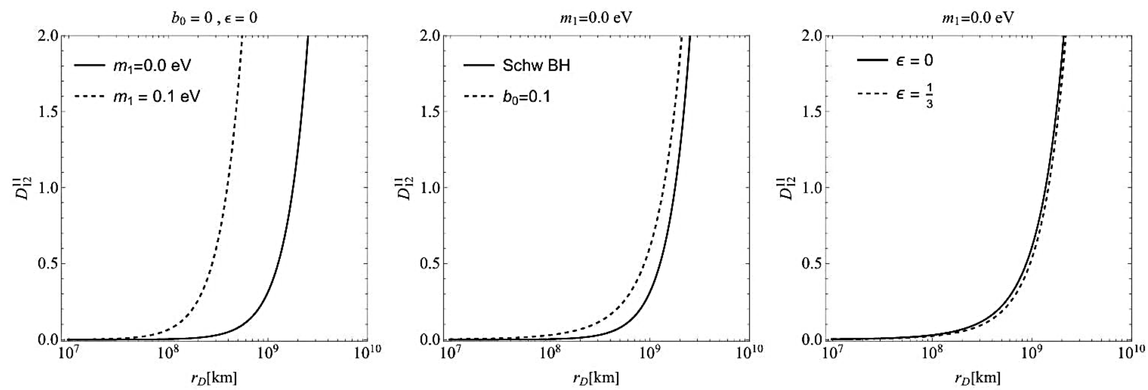

Figure 5. The damping factor D_{12}^{11} as a function of r_D for $\sigma/E_{\mathrm{loc}} = 10^{-13}$. In the left panel, the solid (dashed) line corresponds to $m_1 = 0\mathrm{eV}(m_1 = 0.1\mathrm{eV})$. In the middle panel, the solid and dashed lines correspond to the Schwarzschild and RZ line elements (with $b_0 = 0.1$ and $= 0$), respectively. The right panel is plotted for two different values of the parameter ϵ. Here, $m_1 = 0.0\mathrm{eV}$ and $b_0 = 0.1$. The other parameters are $r_S = 10^5 r_D, \Delta m_{21}^2 = m_2^2 - m_1^2 = 10^{-3}\mathrm{eV}^2$ and $E_{\mathrm{loc}} = 10\mathrm{MeV}$.

Alternatively we can consider another coordinate system $\{x', y'\}$ obtained by rotating the original system (x, y) by an angle φ such that $x' = x\cos\varphi + y\sin\varphi$ and $y' = -x\sin\varphi + y\cos\varphi$. Let us consider the deflection angle δ in the rotated frame as

$$\delta \sim \frac{y'_D - b}{x'_D} = -\frac{4M}{b} = -\frac{2R_x}{b} \tag{29}$$

where $R_x = 2M = r_0(1 + \epsilon)$ and (x'_D, y'_D) is location of the detector. Using the identity $\sin\varphi = b/r_S$, the last equation can be written as

$$(2R_x x_D + b y_D)\sqrt{1-\frac{b^2}{r_S^2}} = b^2\left(\frac{x_D}{r_S}+1\right) - \frac{2R_x b y_D}{r_S} \qquad (30)$$

The solution of Eq.(30) gives the impact parameters in terms of r_S, R_x and the detector's location (x_D, y_D). As an example we shall consider the Sun-Earth system with typical values of the geometrical quantities and assume that the gravitational field of the Sun may be represented by the RZ metric while the Earth's location, $r_D = 10^8$ km, is taken as the detector. We then assume that the source situated behind the Sun at a larger distance $r_S = 10^5 r_D$ and it emits relativistic neutrinos with typical energy $E_0 = 10\text{MeV}$. Now assuming a circular trajectory of the detector around the Sun such that $x_D = r_D \cos\varphi$ and $y_D = r_D \sin\varphi$, we can numerically solve the quartic polynomial given by Eq. (30) and obtain two positive real roots b_1 and b_2 for every φ. In this numerical exercise, the neutrino oscillation probability is calculated only for those value of b_p for which $R_x \ll b_p \ll r_D$. In other words, neutrinos pass far enough from the RZ matter source to consider weak lensing while the detector's location is taken much farther than the impact parameter. The other relevant parameters are $M = 1M_{\odot}$ and $|\Delta m^2| = 10^{-3}\text{eV}^2$. Note that, these numbers are for illustrative purposes only and in a realistic scenario proper numerical values of the geometric parameters of the model must be considered.

The oscillation probabilities for the two-flavor toy model of neutrinos are shown in Figs. 2 and 3. Note that our main aim is to investigate the dependence of the oscillation probability on the parameters of the RZ spacetime. Fig. 2 illustrates the neutrino oscillation probability $\nu_e \to \nu_\mu$ for $b_0 = 0$ (solid line), which corresponds to Schwarzschild, $b_0 = 0.01$ (dotted line) and $b_0 = 0.1$ (dashed line). The figure shows normal hierarchy, i.e. $\Delta m^2 > 0$, in the top panel and inverted hierarchy, i.e. $\Delta m^2 < 0$, in the bottom panel. One can see that the probability depends on the value of the RZ deformation parameter b_0. Here, the mixing angle equals to $\alpha = \pi/6$. In Fig. 3 we illustrate the oscillation probability as a function of azimuthal angle φ. Again it is evident how the parameter b_0 affects the oscillation probability while the effect of ϵ become visible only for b_0 sufficiently large.

It is interesting to notice that there is a degeneracy for different values of the RZ parameters which produce the same oscillation probability. In Fig. 4 we plotted the functions $M(b_0)$ (left panel) and $M(\epsilon)$ (middle panel) obtained implicitly from Eq. (9) for which we obtain the same probability $\mathcal{P}_{\mu\nu}$ at a given value of the angle φ. The right panel of Fig. 4 shows the values of ϵ and b_0 which give the same probability for a given lens mass $M = 2M_{\odot}$ and at a given angle φ.

In the second chapter entitled "**Kerr-Newman-modified-gravity black hole's impact on the magnetic reconnection**" provides information on the magnetic reconnection process as an alternative model of energy extraction from astrophysical compact objects. Moreover, the effect of a given black hole on the magnetic reconnection is discussed.

In Boyer-Lindquist coordinates, the background geometry of the Kerr-Newman-MOG spacetime is given by

$$ds^2 = -\frac{\Delta}{\rho^2}[dt - a\sin^2\theta d\phi]^2 + \rho^2\left[\frac{dr^2}{\Delta} + d\theta^2\right] + \frac{\sin^2\theta}{\rho^2}[(r^2+a^2)d\phi - adt]^2 \quad (31)$$

where

$$\rho^2 = r^2 + a^2\cos^2\theta,\ \Delta = r^2 - 2G_{\mathrm{N}}(1+\alpha)Mr + a^2 + Q^2 + G_{\mathrm{N}}^2\alpha(1+\alpha)M^2 \quad (32)$$

The MOG parameter $\alpha = (G - G_{\mathrm{N}})/G_{\mathrm{N}}$ is a dimensionless measure of the difference between the Newtonian gravitational constant G_{N} and the additional gravitational constant G. The Arnowitt-Deser-Misner (ADM) mass and the angular momentum of the Kerr-MOG black hole are given by $\mathcal{M} = (1+\alpha)M$.
The function Δ can be rewritten in terms of the ADM mass,

$$\Delta = r^2 - 2\mathcal{M}r + a^2 + Q^2 + \frac{\alpha}{1+\alpha}\mathcal{M}^2 \quad (33)$$

where we have set $G_{\mathrm{N}} = 1$ without loss of generality. The spatial locations of the horizons are the roots of Δ as

$$r_H = \mathcal{M} \pm \sqrt{\frac{\mathcal{M}^2}{1+\alpha} - a^2 - Q^2} \quad (34)$$

Notice that the parameters of the Kerr-Newman-MOG spacetime represent a black hole surrounded by an event horizon provided that $\mathcal{M}^2 \geq (1+\alpha)(a^2+Q^2)$.

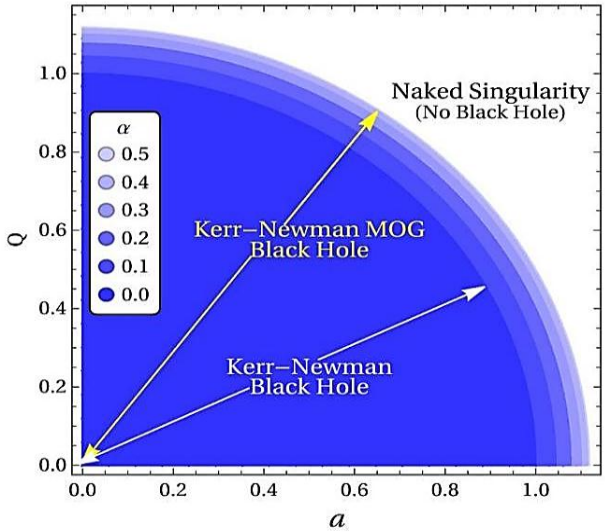

Figure 6. The parameter space plot between the charge parameter Q and the spin parameter a of the Kerr-Newman-MOG BH for the MOG parameter α in the range 0-0.5. Additionally, the parameter M is set to unity throughout.

Here, the inequality corresponds to the case of an extremal black hole, i.e., $a \to a_{\mathrm{ext}}$ that refers to the maximal spin of the black hole. If it is not satisfied the black hole can no longer exist. On the other hand, there also exists another key point that the Kerr-Newman-MOG black hole rotates with the spin greater than that of one for the Kerr black hole due to the effect of MOG parameter α; see in Fig. 6.

Following to Wald we consider the generalized form of vector potential of the electromagnetic field around the Kerr-Newman MOG black hole, which is given by the following

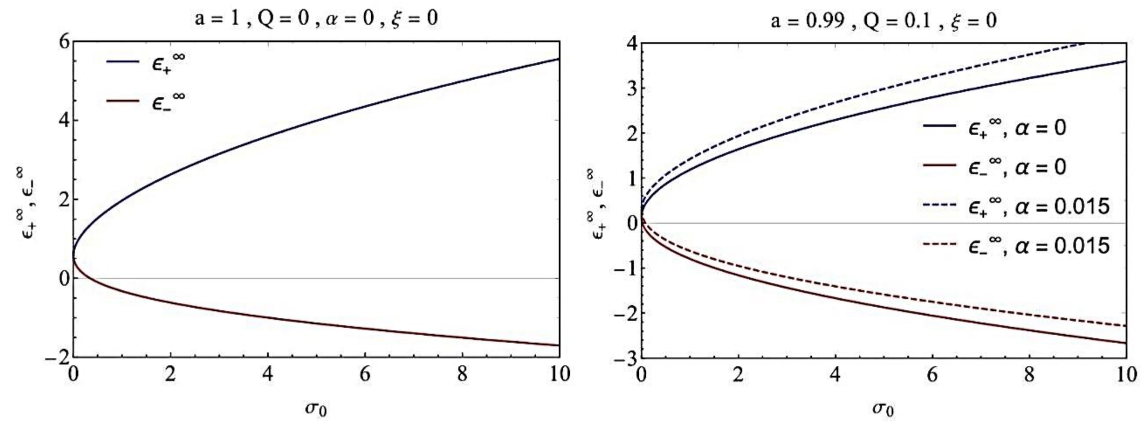

Figure 7. The energies ϵ_+ and ϵ_- per enthalpy at infinity are plotted for various possible cases. Left: ϵ_+ and ϵ_- are plotted for $Q = 0$ and $\alpha = 0$. Right: ϵ_+ and ϵ_- are plotted as a consequence of the presence of the MOG parameter while keeping fixed a and Q. Note that we have set $\xi \to 0$ for maximum energy extraction.

form:

$$A^{\mu} = \left[\frac{Qr}{r^2 - 2f} - aB\left(1 + \frac{f_2 \sin^2\theta}{r^2}\right)\right]\xi_t^{\mu} + \left[-\frac{B}{2}\left(1 + \frac{2f_2}{r^2}\right) + \frac{Qa}{r(r^2 - 2f)}\right]\xi_{\phi}^{\mu} \tag{35}$$

where we have defined $f = f_1 r + f_2$. Note that f_1 and f_2 are defined by

$$f_1 = (1+\alpha)M \; f_2 = -(1+\alpha)(\alpha M^2 + Q^2)/2 \tag{36}$$

Energy extraction through magnetic reconnection process

Black holes are most fascinating objects as that of their rich astrophysical energetic phenomena, i.e., the outflows coming from active galactic nuclei with the energy range of order $E \approx 10^{42} - 10^{47}\,\mathrm{erg/s}$ observed with the help of x rays, γ rays, and very long baseline interferometry observations. These outflows in the form of winds and jets are related to the charged particle motion moving on the accretion disk around the black hole. To explore this energetic phenomenon we analyse the magnetic reconnection process by applying the recently proposed Comisso-Asenjo mechanism, which strongly depends on the frame dragging effect twisting the magnetic field lines around a rapidly spinning black hole. We further consider that the magnetic reconnection takes place in the ergoregion of a rapidly spinning Kerr-Newman MOG black hole so that its rotational energy is driven out efficiently by scenario. For that, we first consider the zero angular momentum observer (ZAMO) frame and then we examine the plasma energy density. For the above mentioned ZAMO frame the line element is written as follows:

$$ds^2 = -d\hat{t}^2 + \sum_{i=1}^{3} \left(d\hat{x}^i\right)^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{37}$$

where we define $d\hat{t}$ and $d\hat{x^{\iota}}$ as follows:

this

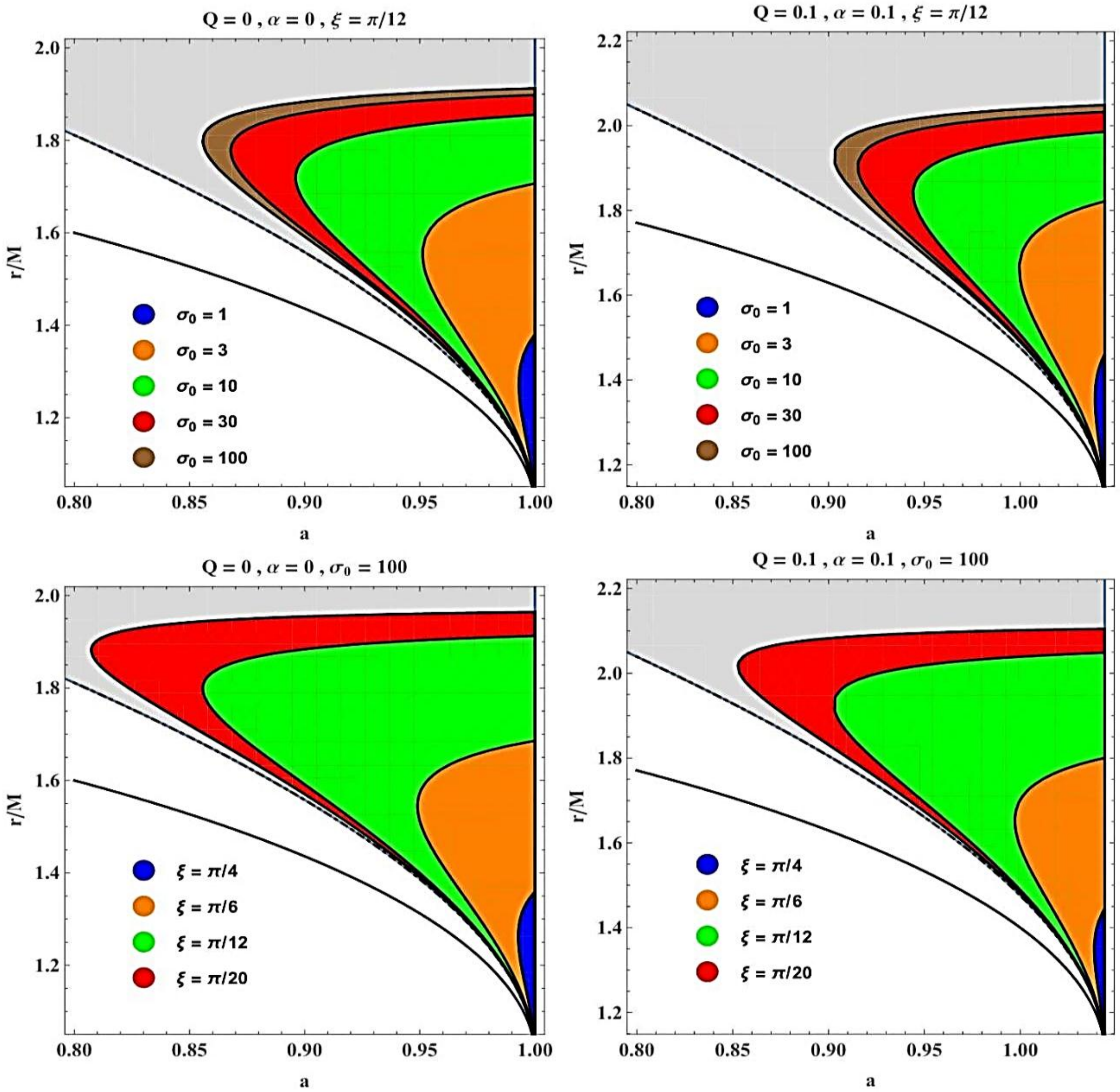

Figure 8. The phase-space regions $\{a, r/M\}$ are plotted for the accelerated plasma energy $\epsilon_+^\infty > 0$ (gray area) and the decelerated plasma energy $\epsilon_-^\infty < 0$ (blue to red areas). Top row, left/right panels: $\{a, r/M\}$ is plotted for various combinations of the plasma magnetization σ_0 in the case of Kerr/Kerr-Newman-MOG black hole cases. Bottom row, left/right panels: $\{a,\ r/M\}$ is plotted for various combinations of the orientation angle ξ in the case of Kerr/Kerr-Newman-MOG black hole cases.

$$d\hat{t} = \alpha dt \text{ and } d\hat{x}^{\iota} = \sqrt{g_{ii}}dx^i - \alpha\beta^\phi dt \tag{38}$$

with α and $\beta^i = (0,0,\beta^\phi)$ that, respectively, refer to the laps function and the shift vector and are written as

$$\alpha = \sqrt{-g_{tt} + \frac{g_{\phi t}^2}{g_{\phi\phi}}} \text{ and } \beta^\phi = \frac{\sqrt{g_{\phi\phi}}\omega^\phi}{\alpha} \tag{39}$$

To be more accurate we define the energy-momentum tensor for the one-fluid approximation of the plasma, and it is given by

$$T^{\mu\nu} = pg^{\mu\nu} + wU^{\mu}U^{\nu} + F^{\mu}_{\delta}F^{\nu\delta} - \frac{1}{4}g^{\mu\nu}F^{\rho\delta}F_{\rho\delta} \quad (40)$$

Note that in the above equation p and w, respectively, refer to the proper plasma pressure and enthalpy density, while U^{μ} and $F^{\mu\nu}$ refer to four-velocity and the tensor of the electromagnetic field. Here we note that the enthalpy density is given by $w = e_{\text{int}} + p$, where the thermal energy density is defined by

$$e_{int} = \frac{p}{\Gamma - 1} + \rho c^2 \quad (41)$$

with Γ and ρ which, respectively, refer to the adiabatic index and the proper mass density. By imposing this condition we further define the relativistic hot plasma with the equation of state. The energy density at infinity can be given by the following relation

$$e^{\infty} = -\alpha g_{\mu 0}T^{\mu 0} \quad (42)$$

Taking this into account, one can write the energy density at infinity as follows:

$$e^{\infty} = \alpha\hat{e} + \alpha\beta^{\phi}\hat{P}^{\phi} \quad (43)$$

where $\hat{e}$ and $\hat{P}^{\phi}$, respectively, define the total energy density and azimuthal component of the momentum density, and they are given by

$$\hat{e} = w\hat{\gamma}^2 - p + \frac{\hat{B}^2 + \hat{E}^2}{2},\ \hat{P}^{\phi} = w\hat{\gamma}^2\hat{v}^{\phi} + (\hat{B} \times \hat{E})^{\phi} \quad (44)$$

where $\hat{v}^{\phi}$ represents the azimuthal component of the plasma velocity at the ZAMO. The Lorentz factor $\hat{\gamma}$ and electric $\hat{E}^{i}$ and magnetic $\hat{B}^{i}$ field components that appear in the above equation are defined by

$$\hat{\gamma} = \hat{U}^0 = \sqrt{1 - \sum_{i=1}^{3} (d\hat{v}^i)^2},\ \hat{B}^i = \epsilon^{ijk}\hat{F}_{jk}/2 \text{ and } \hat{E}^i = \eta^{ij}\hat{F}_{j0} = \hat{F}_{i0} \quad (45)$$

It is worth noting here that the energy density at infinity e^{∞} consists of two parts, i.e., the hydrodynamic and electromagnetic parts with $e^{\infty} = e^{\infty}_{hyd} + e^{\infty}_{em}$ which can be written separately as follows:

$$e^{\infty}_{hyd} = \alpha\hat{e}_{hyd} + \alpha\beta^{\phi}w\hat{\gamma}^2\hat{v}^{\phi},\ e^{\infty}_{em} = \alpha\hat{e}_{em} + \alpha\beta^{\phi}(\hat{B} \times \hat{E})_{\phi} \quad (46)$$

where $\hat{e}_{hyd} = w\hat{\gamma}^2 - p$ and $\hat{e}_{em} = \left(\hat{B}^2 + \hat{E}^2\right)/2$, respectively, refer to the energy densities of the hydrodynamic and electromagnetic fields at the ZAMO. We then need to evaluate the energy density at infinity. For that, we assume that the contribution of the electromagnetic field can be expelled out since its effect is very small in contrast to the hydrodynamic energy density at infinity. However, we note that most of the magnetic field energy can be transferred to the plasma kinetic energy in the magnetic reconnection process. Taking all together we shall further assume that we apply for incompressible and adiabatic plasma for the approximation. In doing so, the energy density at the infinity is then defined by the following form:

$$e^{\infty} = e_{hyd}^{\infty} = \alpha w \hat{\gamma}\left(1 + \beta^{\phi}\hat{v}^{\phi}\right) - \frac{\alpha p}{\hat{\gamma}} \tag{47}$$

Next, one needs to make the magnetic reconnection process the localized one so as to determine it at a small scale. For that, the local rest frame $x^{\mu\mu} = (x^{00}, x^{11}, x'^{2}, x'^{3})$ comes into play because of the bulk plasma that orbits at the equatorial plane around the black hole with the Keplerian frequency/angular velocity Ω_K, which can be defined by

$$\Omega_K = \frac{d\phi}{dt} = \frac{-g_{t\phi,r} + \sqrt{g_{t\phi,r}^2 - g_{tt,r}g_{\phi\phi,r}}}{g_{\phi\phi,r}} \tag{48}$$

The Keplerian frequency for the Kerr-Newman-MOG black hole then reads as follows:

$$\Omega_K = \frac{a(Q^2 - (\alpha+1)(r-\alpha))}{r^4 + a^2(Q^2 - (\alpha+1)(r-\alpha))} + \frac{r^2\sqrt{(\alpha+1)(r-\alpha) - Q^2}}{r^4 + a^2(Q^2 - (\alpha+1)(r-\alpha))} \tag{49}$$

It is worth noting that the direction of x'^{μ} is chosen so that x'^{1} and x'^{3} must be parallel to the radial $x^1 = r$ and the azimuthal $x^3 = \phi$ directions, respectively. Following Eq. (49) we further consider the corotating Keplerian frequency at the ZAMO, and it is given by

$$\hat{v}_K = \frac{d\hat{x}^{\phi}}{d\hat{x}^{t}} = \frac{\sqrt{g_{\phi\phi}}dx^{\phi}/d\lambda - \alpha\beta^{\phi}dx^{t}/d\lambda}{\alpha dx^{t}/d\lambda} = \frac{\sqrt{g_{\phi\phi}}}{\alpha}\Omega_K - \beta^{\phi} \tag{50}$$

One can then obtain the forms of $\hat{v}_K$ and the Lorentz factor $\hat{\gamma}_K = 1/\sqrt{1 - \hat{v}_K^2}$ by imposing the Keplerian frequency Ω_K given by Eq. (48)). The rotation energy of a black hole that can be extracted by the magnetic reconnection process extremely depends upon the plasma dynamics and electromagnetic filed properties. As was mentioned, we assume a one-fluid plasma that obeys adiabatic and incompressible plasma approximation so that the hydrodynamic energy per enthalpy at the infinity reads as follows:

$$\epsilon_{\pm}^{\infty} = \alpha\hat{\gamma}_K\left[\left(1 + \beta^{\phi}\hat{v}_K\right)\sqrt{1+\sigma_0} \pm \cos\xi\left(\beta^{\phi} + \hat{v}_K\right)\sqrt{\sigma_0} - \frac{\sqrt{1+\sigma_0} \mp \cos\xi\,\hat{v}_K\sqrt{\sigma_0}}{4\hat{\gamma}^2(1+\sigma_0 - \cos^2\xi\,\hat{v}_K^2\sigma_0)}\right] \tag{51}$$

with $\sigma_0 = B_0^2/w$ and ξ, which, respectively, refer to the plasma magnetization and the orientation angle between the magnetic field and the outflow plasma directions at the equatorial plane. To extract the black hole rotational energy on the basis of the magnetic reconnection process the hydrodynamic energy would be positive when the plasma is accelerated. In contrast, the energy is negative when it is decelerated near the black hole horizon, similar to what is observed for the Penrose process. Thus, the energy that can be extracted by the magnetic reconnection process should be positive and always much greater than thermal energies and the rest mass of the plasma as well. We shall further assume that the plasma is a relativistic hot plasma that satisfies the condition for the equation of state, i.e., $w = 4p$. With this in view, the accelerated and decelerated energies of the plasma which can be measured at infinity read as follows:

$$\epsilon_{-}^{\infty} < 0 \text{ and } \Delta\epsilon_{+}^{\infty} > 0 \tag{52}$$

where $\Delta\epsilon_{+}^{\infty}$ is given by

$$\Delta\epsilon_{+}^{\infty} = \epsilon_{+} - \left(1 - \frac{\Gamma}{\Gamma - 1}\frac{p}{w}\right) > 0 \tag{53}$$

Taking the polytropic index as $\Gamma = 4/3$ allows one to have the form as $\Delta\epsilon_{+}^{\infty} = \epsilon_{+}^{\infty} > 0$ for the relativistic hot plasma.

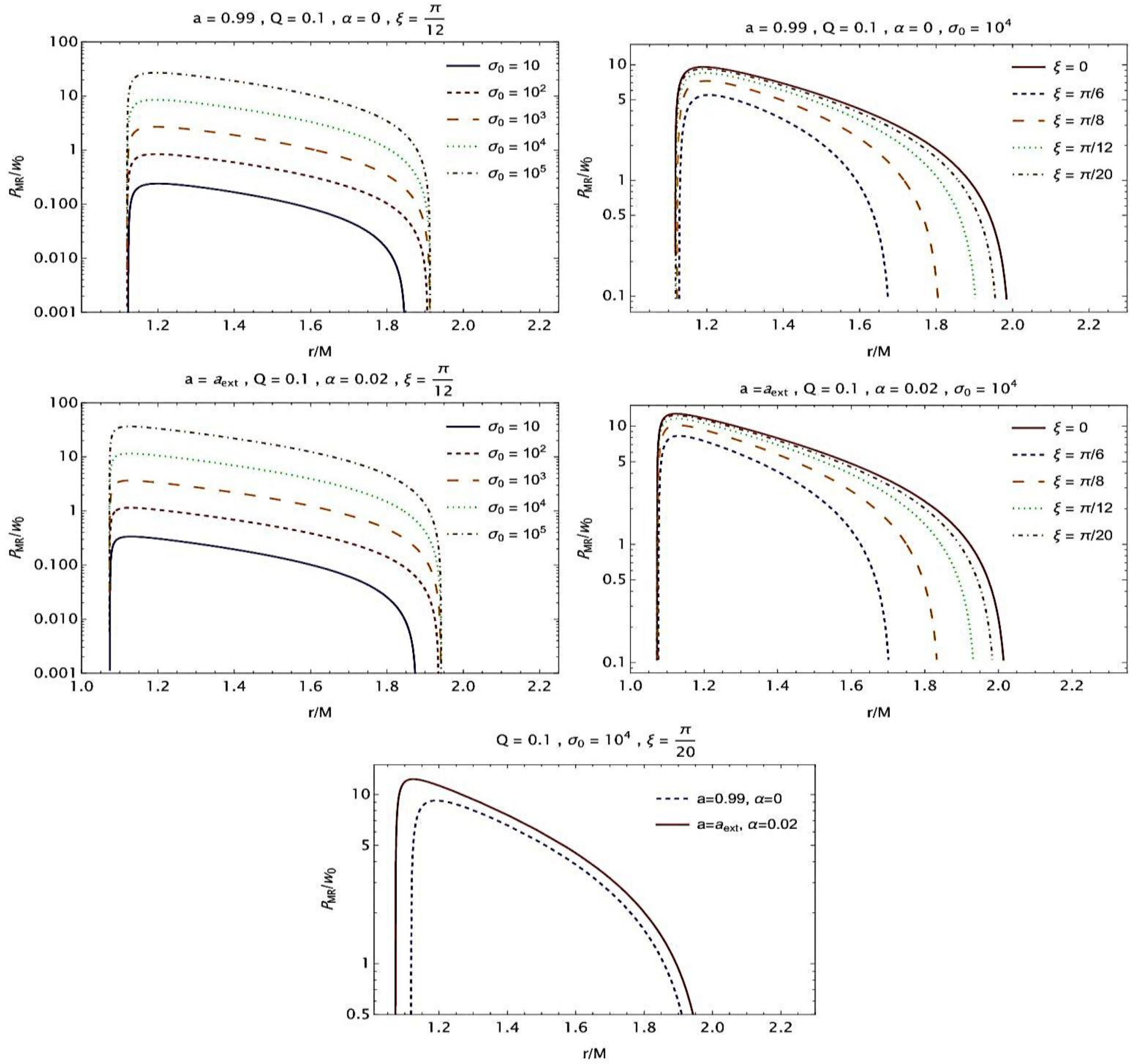

Figure 9. The power P_{MR}/w_0 of the magnetic reconnection is plotted for a rapidly spinning Kerr-Newman-MOG black hole. Top row, left: P_{MR}/w_0 is plotted for various combinations of the plasma magnetization σ_0 while keeping fixed a, Q, and ξ. Top row, right: P_{MR}/w_0 is plotted for various combinations of the orientation angle ξ while keeping fixed a, Q, and σ_0. Middle row similar to what is observed in the top row, P_{MR}/w_0 is plotted as a consequence of the presence of MOG parameter α. Bottom row: P_{MR}/w_0 is plotted for Kerr and Kerr-Newman-MOG black hole cases while keeping fixed $Q = 0.1, \sigma_0 = 10^4$, and $\xi = \pi/20$. Note that an extremal value of the spin parameter refers to $a_{ext} = (1 + \alpha - Q^2)^{1/2}$.

We now analyze the accelerated ϵ_+^∞ and decelerated ϵ_-^∞ energies of the plasma to explore the remarkable aspects of energy extraction through the magnetic reconnection process for the Kerr-Newman-MOG black hole. However, the analytic forms of ϵ_+^∞ and ϵ_-^∞ energies turn out to be very long and complicated expressions for explicit display. In Fig. 7, we therefore show the plasma energies ϵ_+^∞ and ϵ_-^∞ as a function of the plasma magnetization parameter for various possible cases.

Figure 8 reflects the role of the magnetization parameter σ_0 (top row, left and right panels) and the orientation angle ξ (bottom row, left and right panels) on the regions of the phase space ($a, r/M$). The most important key point to be noted here is that the combined effect of black hole charge and MOG parameter can extend the phase space for the energy extraction condition, i.e., $\delta\epsilon_+^\infty > 0$ (gray area), as seen in both top and bottom right panels of Fig. 8.

Next, we investigate the power and energy efficiency via the magnetic reconnection process for the Kerr-Newman-MOG black hole.

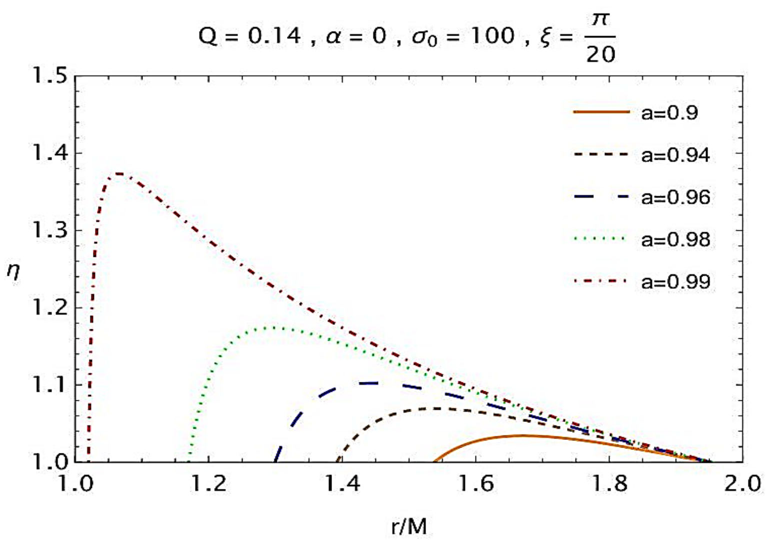

Figure 10. The magnetic reconnection efficiency against the location r/M for various combinations of spin parameter (left) and MOG parameter α (right) in the case of fixed black hole charge Q. Here we have set $\sigma_0 = 100$ for the plasma magnetization $\xi = \pi/20$ for the orientation angle the reconnecting magnetic can have.

Power and magnetic reconnection efficiency

In this abstract, we consider the power and the energy efficiency via the magnetic reconnection process and resort to the numerical evaluation of these quantities for the Kerr-Newman-MOG black hole. It is worth noting that energy efficiency and power are strongly related to the negative energy of decelerated plasma which is absorbed by a black hole in the unit time. We first turn to the power of the energy extraction in which it is envisaged that the power that is extracted from a black hole due to the escaping plasma can be defined by

$$P_{MR} = -\epsilon_-^\infty w_0 A_{in} U_{in} \tag{54}$$

where U_{in} defines the regime, i.e., $U_{\text{in}} = \mathcal{O}(10^{-1})$ refers to the collisionless regime, while $U_{\text{in}} = \mathcal{O}(10^{-2})$ to the collisional regime. Also, A_{in} given in the above expression defines the cross sectional area for the inflowing and is evaluated as $A_{in} = \left(r_E^2 - r_{ph}^2\right)$ for rotating black holes.

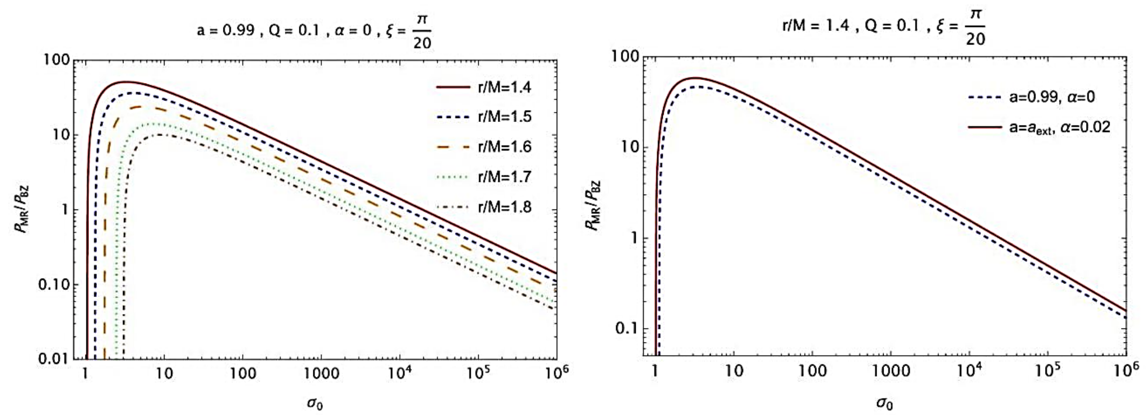

Figure 11. The ratio of power P_{MR}/P_{BZ} against the plasma magnetization σ_0. Left: P_{MR}/P_{BZ} is plotted for various combinations of location r/M for fixed values of black hole parameters and the orientation angle. Right: P_{MR}/P_{BZ} is plotted for two values of MOG parameter α and corresponding values of spin parameter.

Let us then turn to another key aspect of energy extraction via magnetic reconnection. To understand how magnetic reconnection is an effective model, one needs to analyze the energy release that can be extracted by the model considered here. Therefore, we need to evaluate the total amount of energy released by the Kerr-Newman-MOG black hole. We first would like to emphasize that the magnetic field energy during the magnetic reconnection, after being distributed, consists of two parts of energies, i.e., the decelerated plasma and the accelerated plasma energies. The former one having negative energy is absorbed by the black hole, while the latter one with arbitrarily high positive energy comes out toward to larger r distances from the black hole. With this in view, the energy efficiency of the plasma through the magnetic reconnection can generally be defined by

$$\eta = \frac{\epsilon_+^\infty}{\epsilon_+^\infty + \epsilon_-^\infty} \tag{55}$$

where ϵ_+^∞ and ϵ_-^∞, respectively, define the accelerated and decelerated plasma energies, as mentioned above. The key point to be noted here is that for the plasma energy to be released from a black hole the condition $\eta = \epsilon_+^\infty/(\epsilon_+^\infty + \epsilon_-^\infty) > 1$ must always be satisfied for the energy efficiency. We further then analyze the energy efficiency released by the magnetic reconnection.

Finally we turn to compare the power of the magnetic reconnection and Blandford-Znajek mechanisms in order to estimate the rate of energy extraction under the fast magnetic reconnection. To that, we first consider the rate of energy extraction for the BZ mechanism at the horizon, and it is defined by

$$P_{\mathrm{BZ}} = \frac{\kappa}{16\pi}\Phi_{BH}^2\Omega_H^2\left[1 + \chi_1\Omega_H^2 + \chi_2\Omega_H^4 + \mathcal{O}(\Omega_H^6)\right] \tag{56}$$

where Φ_{BH} and Ω_H, respectively, denote the magnetic flux and the angular velocity Ω_H at the horizon which is given by

$$\Omega_H = \frac{a}{2\mathcal{M}r - Q^2 - \frac{\alpha}{1+\alpha}\mathcal{M}^2} \tag{57}$$

while κ, χ_1, and χ_2 refer to numerical constants. Here we note that κ has a relation with geometric configuration. The magnetic flux is then defined by $\Phi_{BH} = \frac{1}{2}\int_\theta \int_\phi |B^r| dA_{\theta\phi}$, and we shall for simplicity assume $\Phi_{BH} \sim 2\pi B_0 r_H^2 \sin\xi$ for further analysis. Here we do, however, note that for the magnetic field configuration the orientation angle ξ can be regarded as a good estimate only at low latitudes. In fact, one needs to evaluate the magnetic flux Φ_{BH} with the magnetic field configuration at all latitudes. Taking all together the ratio of these two powers P_{MR}/P_{BZ} can then be written by

$$\frac{P_{MR}}{P_{BZ}} = \frac{-4\epsilon_-^\infty w_0 A_{in} U_{in}}{\pi\kappa\Omega_H^2 r_H^4 \sigma_0 \sin^2\xi (1 + \chi_1\Omega_H^2 + \chi_2\Omega_H^4)} \tag{58}$$

where we note that the values of these numerical constants have been approximated as $\chi_1 \approx 1.38$ and $\chi_2 \approx -9.2$, while $\kappa \approx 0.044$ due to the magnetic field geometry.

In Fig. 11 we show the behavior of P_{MR}/P_{BZ} against the plasma magnetization for various combinations of the location r/M while keeping black hole parameters fixed. From the left panel of Fig. 11, the height of the ratio of powers decreases and the curves shift right to larger σ_0 as the location r/M increases. It does, however, satisfy $P_{MR}/P_{BZ} > 1$ that makes the magnetic reconnection significantly more efficient than the one for the BZ mechanism. Similar to what is observed in the left panel of Fig. 11, the right panel reflects the impact of the combined effect of black hole charge and MOG parameter on the power ratio P_{MR}/P_{BZ} for different possible locations r/M. As the MOG parameter increases, the curves of the power ratio shift left to smaller σ_0 for both locations, as seen in the right panel of Fig. 11. It can be seen that the shape of the ratio P_{MR}/P_{BZ} increases due to the combined effect of black hole charge and MOG parameter, thus resulting in increasing the efficiency of magnetic reconnection and allowing to mine out more energy from the black hole than for the BZ mechanism.

In the third chapter of the thesis, "**Electric Penrose process and the accretion disk around a 4D charged Einstein-Gauss-Bonnet black hole**", another alternative model, the electric Penrose process, is studied. The extraction of energy by charged particles from the charged black hole was analyzed.

The line element describing the static black hole spacetime in 4D EGB theory is given by

$$ds^2 = -\mathrm{f}(r)dt^2 + \frac{1}{\mathrm{f}(r)}dr^2 + r^2 d\Omega^2 \tag{59}$$

with metric function

$$\mathrm{f}(r) = 1 + \frac{\mathrm{r}^2}{2\alpha}\left(1 - \sqrt{1 + \frac{4\alpha}{\mathrm{r}^2}\left(\frac{2\mathrm{M}}{\mathrm{r}} - \frac{\mathrm{Q}^2}{\mathrm{r}^2}\right)}\right)$$

. Where M is the total mass and Q the black hole electric charge. We write the conservation laws for particles around this spacetime, derive the energy efficiency expression and express it in the corresponding graphs.

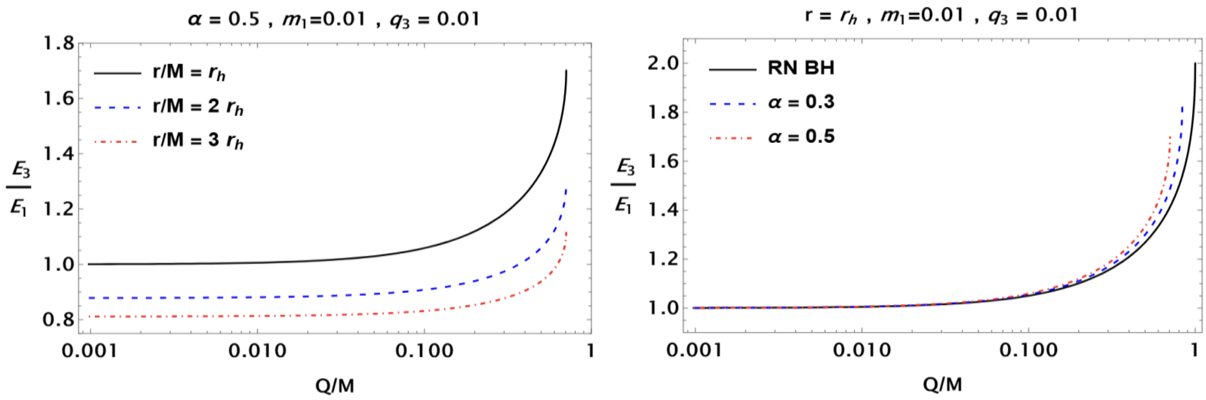

Figure 12. The energy ratio (i.e., the energy efficiency of the electric PP) between ionized and neutral particles as a function of black hole charge. Left panel: the energy efficiency is plotted for different values of the ionization points r/M. Right panel: the energy efficiency is plotted for different values of the parameter α.

It should be noted that in the dissertation the efficiency of the electric Penrose process is greater than the efficiency of the ordinary Penrose process.

CONCLUSIONS

The following conclusions have been presented on the basis of research carried out on the topic of "Observational properties of black hole mimickers and alternative models to their energetics":

1. It has been demonstrated that the accelerated and decelerated energies per enthalpy grow as a consequence of the combined effect of the MOG parameter and black hole charge leading to arbitrarily high energy that can be extracted out by the magnetic reconnection. It has been also found that the combined effect of black hole charge and MOG parameter can extend the phase space for the energy extraction condition which leads to arbitrarily high energy driven out from the Kerr-Newman-MOG black hole via magnetic reconnection.

2. It has been shown that the combined effect of black hole charge and MOG parameter leads to high power through the magnetic reconnection as compared to the Kerr black hole. It has been also found that the combined effect of MOG parameter and black hole charge play important role in approaching high energy efficiency: the efficiency of energy extraction reaches up to its possible maximum due to the presence of attractive MOG parameter.

3. It has been shown that the ratio of efficiencies due to magnetic reconnection and Blandford-Znajek mechanism increases due to effect of black hole charge and MOG parameter. It has been stated that the magnetic reconnection is significantly more efficient than for the Blandford-Znajek mechanism: as a result the efficiency increases as compared to the Kerr black hole. It has been concluded that the MOG parameter can cause even more fast spin that strongly affects the reconfiguration of magnetic field lines due to the frame dragging effect.

4. It has been shown that the probability of neutrino oscillations depends on the values of the parameters in the Rezzolla-Zhidenko metric which characterize the deviation of the geometry from Schwarzschild. It has been obtained that it may be possible to determine features of the geometry from the observations of neutrinos.

5. It has been shown that beyond a certain length, neutrinos lose coherence and the decoherence length has insignificant effects from the deformation parameters of the Rezzolla-Zhidenko metric as compared to the absolute neutrino masses.

НАУЧНЫЙ СОВЕТ DSc.03/31.03.2022.T/FM.10.04 ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ ФУНДАМЕНТАЛЬНЫХ И ПРИКЛАДНЫХ ИССЛЕДОВАНИЙ, «ТИИИМСХ» НАЦИОНАЛЬНЫЙ ИССЛЕДОВАТЕЛЬСКИЙ УНИВЕРСИТЕТ

ИНСТИТУТ ФУНДАМЕНТАЛЬНЫХ И ПРИКЛАДНЫХ ИССЛЕДОВАНИЙ

АЛЛОКУЛОВ МИРЗАБЕК УЛУГБЕК УГЛИ

НАБЛЮДАТЕЛЬНЫЕ СВОЙСТВА ИМИТАТОРОВ ЧЕРНЫХ ДЫР И АЛЬТЕРНАТИВНЫЕ МОДЕЛИ ИХ ЭНЕРГЕТИКИ

01.03.01 – Астрономия
01.04.02 – Теоретическая физика

АВТОРЕФЕРАТ
ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD)
ПО ФИЗИКО –МАТЕМАТИЧЕСКИМ НАУКАМ

Ташкент – 2024

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве высшего образования, науки и инноваций Республики Узбекистан под номером B2024.1.PhD/FM1027.

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Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице Научного совета (www.ifar.uz) и Информационно-образовательном портале «Ziyonet» (www.ziyonet.uz).

Научные руководители:	**Ахмедов Бобомурат Жураевич** доктор физико-математических наук, академик
	Шайматов Санжар Рузимуротович доктор физико-математических наук
	Даниеле Малафарина доктор физико-математических наук, профессор
Официалные оппоненты:	**Абишев Медеу Ержанович** доктор физико-математических наук, академик
	Холиков Шукиржон Содикович доктор физико-математических наук
Ведущая организация:	**Самаркандский государственный университет имени Шарофа Рашидова**

Зашита диссертации состоится **«_»_________ 202_ года в ____ часов** на заседании Научного Совета **DSc.03/31.03.2022.T/FM.10.04** по защитедиссертаций на соискание ученых степеней при Институте фундаментальных и прикладных исследований, “ТИИИМСХ” Национальный Исследовательскийуниверситет по адресу: 100000, г. Ташкент, ул. Кары Ниязий 39, Институт фундаментальных и прикладных исследований, Зал 108; Тел.: 71 237-09-61; email: info@ifar.uz.

С диссертацией можно ознакомиться в Информационно-ресурсном центре при Институте фундаментальных и прикладных исследований, “ТИИИМСХ” Национальный Исследовательский университет(регистрационный номер) (Адрес: 100000, г. Ташкент, ул. Кары Ниязий 39, Институт фундаментальных и прикладных исследований, Зал 205; Тел.: 71 237-09-61).

Автореферат диссертации разослано «________»___________202_ г.
(протокол рассылки № 49 от________________202_ г.).

Б.А. Тошматов
Заместитель председателя Научного совета по присуждению ученых степеней, д.ф.-м.н.

Ш.Н. Мардонов
Ученый секретарь Научного совета по присуждению ученых степеней д.ф.-м.н.

Б.М. Нарзиллоев
Преседатель Научного семинара при Научном совете по присуждению ученых степеней, д.ф.-м.н.

ВВЕДЕНИЕ (Аннотация к представлению)

Целью исследования является изучение наблюдательных свойств подражателей (имитаторов) черных дыр (ЧД) и поиск альтернативных моделей их энергетики.

Задачи исследования:

проанализировать влияние вращающейся черной дыры Керра-Ньюмана-МОГ на магнитное пересоединение через механизм Комиссо-Асенжо;

изучить извлечение энергии ЧД процессом магнитного пересоединения, непрерывно происходящим внутри эргосферы за счет вращения черной дыры;

исследовать энергетическую эффективность извлечения энергии и мощности ЧД в зависимости от намагниченности плазмы, ориентации магнитного поля, спина черной дыры и параметров МОГ путем наложения всех необходимых условий;

исследовать ускоренные и замедленные энергии плазмы для извлечения энергии ЧД посредством магнитного пересоединения;

изучить область фазового пространства ЧД на предмет выполнения требуемого условия отбора энергии;

изучить мощность и энергетическую эффективность быстро вращающейся черной дыры Керра-Ньюмана, чтобы понять, насколько эффективно магнитное пересоединение по сравнению со случаем черной дыры Керра;

сделать оценку скорости извлечения энергии при первом магнитном пересоединении путем сравнения мощности магнитного пересоединения и механизмов Бландфорда-Знаека;

проанализировать поведение КПД в зависимости от намагниченности плазмы для различных возможных случаев;

изучить колебания ароматов нейтрино, распространяющихся в пространстве-времени Рецоллы-Жиденко, и определить, как обнаружение нейтрино в принципе можно использовать для получения ограничения на параметры геометрии пространства, в которой они путешествуют.

Объектом исследования являются астрофизические компактные объекты, динамика частиц, альтернативные модели энергетики астрофизических компактных объектов.

Предметом исследования являются наблюдательные свойства подражателей (имитаторов) черных дыр, альтернативные модели их энергетики, динамика частиц вокруг астрофизических компактных объектов, аналитические и численные методы решения дифференциальных уравнений движения частиц.

Методами исследования являются методы теоретической физики и астрофизики, современные методы теоретической астрофизики и математической физики, аналитические и численные методы решения дифференциальных уравнений, связанных с динамикой поля и частиц.

Научная новизна исследования заключается в следующем:

Энергоэффективность и мощность отбора энергии исследованы путем наложения всех необходимых условий в зависимости от намагниченности плазмы, направления магнитного поля, спина черной дыры и параметров МОГ.

Впервые проанализировано влияние вращающейся черной дыры Керра-Ньюмана-МОГ на магнитное пересоединение посредством механизма Комиссо-Асенхо.

Мощность и энергетическая эффективность быстро вращающихся черных дыр Керра-Ньюмана-МОГ были исследованы, чтобы понять, насколько эффективно магнитное пересоединение по сравнению со случаем черной дыры Керра.

Впервые сравнивая мощность магнитного пересоединения и механизмов Бландфорда-Знаека, была оценена скорость извлечения энергии при быстром магнитном пересоединении.

Впервые были изучены колебания ароматов нейтрино, распространяющихся в пространстве-времени Резцоллы-Жиденко, и определено, как обнаружение нейтрино в принципе можно использовать для ограничения параметров геометрии пространства, в которой они путешествуют.

Практические результаты исследования, следующие:

Проанализировано влияние вращающейся черной дыры Керра-Ньюмана-МОГ на магнитное пересоединение посредством механизма Комиссо-Асенхо.

Энергоэффективность и мощность отбора энергии исследованы путем наложения всех необходимых условий в зависимости от намагниченности плазмы, направления магнитного поля, спина черной дыры и параметров МОГ.

Мощность и энергетическая эффективность быстро вращающихся черных дыр Керра-Ньюмана-МОГ были исследованы, чтобы понять, насколько эффективно магнитное пересоединение по сравнению со случаем черной дыры Керра.

Сравнивая мощность магнитного пересоединения и механизмов Бландфорда-Знаека, была оценена скорость извлечения энергии при быстром магнитном пересоединении.

Изучены колебания ароматов нейтрино, распространяющихся в пространстве-времени Резцоллы-Жиденко, и определено, как обнаружение нейтрино в принципе можно использовать для ограничения геометрии, в которой они путешествуют.

Достоверность результатов исследования обеспечивается за счет применения создал современные методы математической физики, вычислительной математики и релятивистской астрофизики. Результаты были получены строго в математических рамках общей теории относительности и теоретической физики. Также применяются современные численные и аналитические методы расчета, результаты которых сравниваются с существующими данными наблюдений и выводами других исследователей. Структурированные выводы презентации соответствуют фундаментальным принципам астрофизики, касающимся компактных объектов.

Научная и практическая значимость результатов исследования. Научная значимость результатов исследования состоит в том, что выбранная метрика Резцола-Жиденко включает в себя все сферически-симметричные статические метрики.

Практическая значимость результатов исследования заключается в том, что предложенная модель поражателей (имитаторов) черных дыр может сыграть важную роль в объяснении высокоэнергетических процессов.

Внедрение результатов исследования. На основе энергетики астрофизически компактных объектов:

научные результаты, полученные по энергетике астрофизически компактных объекты использованы в работах зарубежных исследователей, в зарубежных журналах с высоким импакт-фактором (Physical Review D, Volume 110, article id. 063003 Web-Sc, IF: 5.407 va Physical Review D, Volume 109, article id. 084066 Web-Sc, IF: 5.407).

Публикация результатов исследований. Результаты исследования для степени доктора философии представлены в 14 рецензируемых статьях, опубликованных в престижных научных журналах, рекомендованных Высшей аттестационной комиссией при Министерстве высшего образования, науки и инноваций Республики Узбекистан.

Структура и объем диссертации. Диссертация состоит из введения, трех глав, заключения и библиографии. Объем диссертации составляет 120 страниц.

ВЫВОДЫ

На основе исследований, проведенных по теме «Наблюдательные свойства подражателей (имитаторов) черных дыр и альтернативные модели их энергетики», были представлены следующие выводы:

1. Продемонстрировано, что ускоренная и замедленная энергии на энтальпию растут вследствие совместного воздействия параметра МОГ и заряда черной дыры, что приводит к сколь угодно высокой энергии, которая может быть извлечена путем магнитного пересоединения. Также было обнаружено, что совокупный эффект заряда черной дыры и параметра МОГ может расширить фазовое пространство для условия извлечения энергии, что приводит к сколь угодно высокой энергии, извлекаемой из черной дыры Керра Ньюмана-МОГ посредством магнитного пересоединения.
2. Показано, что совместное влияние заряда черной дыры и параметра МОГ приводит к большей мощности за счет магнитного пересоединения по сравнению с черной дырой Керра. Также обнаружено, что совместное влияние параметра МОГ и заряда черной дыры играет важную роль в достижении высокой энергетической эффективности: эффективность извлечения энергии достигает возможного максимума благодаря наличию притягивающего параметра МОГ. Получены аналитические выражения для компонент тензора энергии-импульса самогравитирующего скалярного поля. Показано, что при наличии фантомного поля решение удовлетворяет условию нулевой энергии, а при наличии гравитирующего скалярного поля оно не удовлетворяет условию нулевой энергии.
3. Показано, что соотношение эффективностей за счет магнитного пересоединения и механизма Бландфорда-Знаека увеличивается за счет влияния заряда черной дыры и параметра МОГ. Установлено, что магнитное пересоединение существенно более эффективно, чем для механизма Бландфорда-Знаека: в результате эффективность увеличивается по сравнению с черной дырой Керра. Сделан вывод, что параметр МОГ может вызвать еще более быстрое вращение, что сильно влияет на реконфигурацию силовых линий магнитного поля из-за эффекта увлечения инерциальных систем отсчета.
4. Показано, что вероятность нейтринных осцилляций зависит от значений параметров метрики Рецщоллы-Жиденко, характеризующих отклонение геометрии от геометрии Шварцшильда. Получено, что по наблюдениям нейтрино можно определить особенности геометрии.
5. Показано, что за пределами определенной длины нейтрино теряют когерентность, а длина декогеренции незначительно влияет на параметры деформации метрики Рецщоллы-Жиденко по сравнению с абсолютными массами нейтрино.

E’LON QILINGAN ISHLAR RO‘YXATI
СПИСОК ОПУБЛИКОВАННЫХ РАБОТLIST
OF PUBLISHED WORKS

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II bo‘lim (part II; II часть)

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14. Alloqulov M., Particle dynamics around black hole in Einstein-Maxwell-Scalar gravity // Involta Innovation scientific journal Volume 1, 5(2022).

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“DAVR MATBUOT SAVDO” MCHJ
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